1. There are three forms of line integrals we’ve addressed:

(I) \[ \int_C f \, ds = \int_a^b f(c(t))\|c'(t)\| \, dt \]

(II) \[ \int_C f \, dx = \int_a^b f(c(t))x'(t) \, dt \]

or \[ \int_C f \, dy = \int_a^b f(c(t))y'(t) \, dt \]

(III) \[ \int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(c(t)) \cdot c'(t) \, dt \]

Decide whether to use approach (I), (II) or (III) when you want to

(a) integrate the function \( g(x, y, z) = 3xy - y^2z^5 \) with respect to arclength along the path \( c(t) = (t, 2, t - t^3) \) from the point \((-1, 2, 0)\) to \((0, 2, 0)\);

(b) integrate the function \( f(x, y) = x^2y \) with respect to \( x \) along the path \( x = y^3 \) from \((0, 0)\) to \((2, 8)\);

(c) integrate \( \mathbf{F}(x, y, z) = (z, xy, 2y) \) along the path \( c(t) = (e^t, \ln t, te^t) \) from \( t = 0 \) to \( t = 2 \);

(d) integrate \( f(x, y) = (x^2y, y) \) along the path \( c(t) = (\cos t, \sin t) \) from \( t = 0 \) to \( t = 2\pi \);
(e) integrate $F(x, y, z) = y$ with respect to arclength along the path $c(t) = (t^2, 1, 3t^3)$ from $(0, 1, 0)$ to $(1, 1, 3)$.

2. For each of the following integrals, determine whether to use the Fundamental Theorem of Line Integrals, Green’s Theorem, or neither.

(a) integrate the field $F(x, y, z) = \langle y^2z + y\cos(xy), 2xyz + x\cos(xy), xy^2 \rangle$ along the curve $x^2 + y^2 = 9$ from $(0, 3)$ to $(-3, 0)$

(b) integrate the field $F(x, y) = \langle xy, xy^2 \rangle$ along the curve $x = y^2 + 2$ from $(3, -1)$ to $(2, 0)$

(c) integrate the field $F(x, y, z) = yi + xzj + k$ along the line segments connecting $(0, 0)$ to $(2, 1)$ to $(0, 3)$ to $(0, 0)$

3. Kermit is hopping between two lily pads at $(0, 0, 0)$ and $(2, 4, 0)$. Gravity together with the wind over his swamp give a total force field of $(2xy, x^2, -2)$ through which he hops. The path Kermit plans to follow is given by $c_1(t) = (t, 2t, 2t - t^2)$.

(a) Find how much work Kermit will do by integrating the force field along his path.
(b) There’s a tasty bug out of reach of Kermit’s intended path, so instead he jumps extra high to catch it. His new path is

\[ \mathbf{c}_2(t) = (t, 2t, 4t - 2t^2). \]

If eating the bug will give Kermit 2 units of energy, is it worth it for him to follow the second path?

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(c) Notice that the force field \( \mathbf{F}(x, y, z) = (2xy, x^2, -2) \) is actually the gradient of a function \( f(x, y, z) \). Find that function \( f \).
(d) Now compute the work again using the Fundamental Theorem of Line Integrals, which says that

$$\int_C \nabla f \cdot ds = f(c(b)) - f(c(a)),$$

where the path $c(t)$ goes from $t = a$ to $t = b$. Use this to confirm your solutions in parts (a) and (b).