Show all work for each problem so that partial credit may be given. Answers without supporting work will receive no credit. Clearly indicate your answers. Make sure that you have 7 problems on your test. A scientific calculator may be used.
1. A space pioneer is in trouble near the sunny side of Mercury. The temperature of the spaceship’s hull at point \((x, y, z)\) will be given by

\[
T(x, y, z) = e^{-xy^2z},
\]

where \(x, y,\) and \(z\) are all measured in meters. The ship is currently at \((1, 2, 1)\).

(a) In what direction should the spaceship travel in order to experience the fastest decrease in temperature? (Express this direction as a unit vector.)

(b) What is the rate of change of the temperature if the ship proceeds in that direction?
2. Consider the surface \( x^2 + y^2 + 2x - 4y - z = -7 \).
   (a) Identify this surface.

   (b) Find a parametric equation for the normal line to this surface at the point \((x, y) = (-2, 2)\).

   (c) Find an expression for the tangent plane at the point \((-2, 2)\). Write the plane equation in the form \(x + Ay + Bz + C = 0\).
3. Determine whether the following limits exist.

(a) \[ \lim_{(x,y) \to (0,0)} \frac{3x^2y}{x^3 + y^3} \]

(b) \[ \lim_{(x,y) \to (0,0)} \frac{3x^2y}{x^2 + y^2} \]
4. Measuring the volume of a rectangular box, we find that the sides have lengths approximately 10 cm, 5 cm, and 8 cm. In each of these measurements, we estimate an error of about .1 cm. Use differentials to determine the error we should expect when we compute the volume of the box.
5. A function \( f(x, y) \) is harmonic if it satisfies the Laplace equation

\[
\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.
\]

Show that \( f(x, y) = x^3 - 3xy^2 \) is harmonic.
6. Use Lagrange Multipliers to find the maximum and minimum values of the function

\[ f(x, y) = 2x^2 - 8y + 5, \]

subject to the constraint \( x^2 + 4y^2 = 4. \)