1. (a) Find \( f_y(1, 2) \), where

\[
F(x, y) = \frac{3xy}{\sqrt{x + 2y}}.
\]

\[
f_y(1, 1) = \frac{\sqrt{x + 2y} \cdot 3x - 3xy \cdot \frac{1}{2}(x + 2y)^{-1/2} \cdot 2}{x + 2y} \bigg|_{(1,2)}
\]

\[
= \frac{3\sqrt{5} - \frac{6}{\sqrt{5}}}{5} = \frac{9}{5\sqrt{5}} = \frac{9\sqrt{5}}{25}
\]

(b) Let \( h(x, y) = x \sin(xy) \). Find \( \frac{\partial^2 h}{\partial x \partial y} \) in two different ways.

First method:

\[
h_x = xy \cos(xy) + \sin(xy)
\]

\[
h_{xy} = -x^2y \sin(xy) + x \cos(xy) + x \cos(xy)
\]

\[
= 2x \cos(xy) - x^2y \sin(xy)
\]

Second method:

\[
h_y = x^2 \cos(xy)
\]

\[
h_{xy} = h_{yx} = 2x \cos(xy) - x^2y \sin(xy)
\]

2. Is the following function continuous at \((0, 0)\)? Prove it.

\[
g(x, y) = \begin{cases} 
\frac{2x^3y \sin(xy)}{(x^2 + y^2)^{3/2}} & \text{if } (x, y) \neq (0, 0) \\
0 & \text{if } (x, y) = (0, 0)
\end{cases}
\]

In order to show that this function is continuous at \((0, 0)\), I will show that

\[
\lim_{{(x,y) \to (0,0)}} g(x, y) = 0 = g(0, 0).
\]

\[
\left| \frac{2x^3y \sin(xy)}{(x^2 + y^2)^{3/2}} \right| = \frac{2|x|^3y|\sin(xy)|}{(x^2 + y^2)^{3/2}}
\]
\[
\leq \frac{2(\sqrt{x^2 + y^2})^3 \sqrt{x^2 + y^2} \cdot 1}{(x^2 + y^2)^{3/2}} = 2\sqrt{x^2 + y^2} \longrightarrow 0 \text{ as } (x, y) \to (0, 0)
\]

By the Squeeze Theorem,

\[
\lim_{(x, y)\to(0,0)} g(x, y) = \lim_{(x, y)\to(0,0)} \frac{2x^3y \sin(xy)}{(x^2 + y^2)^{3/2}} = 0.
\]

Since \(\lim_{(x, y)\to(0,0)} g(x, y) = 0 = g(0, 0)\), \(g(x, y)\) is continuous at \((0, 0)\).

3. Consider the function

\[f(x, y) = \sqrt{36 - 4x^2 - 9y^2}.
\]

(a) What is the domain of this function? Sketch the domain in the \(xy\)-plane.

This function is defined when \(36 - 4x^2 - 9y^2 \geq 0\), or \(\frac{x^2}{9} + \frac{y^2}{4} \leq 1\).

\[D = \{(x, y) \in \mathbb{R}^2 : \frac{x^2}{9} + \frac{y^2}{4} \leq 1\}
\]

(b) What is the range of this function?

\[R = \{z \in \mathbb{R} : 0 \leq z \leq 6\}
\]

(c) The graph of this function is part of a quadric surface. What is the name of that surface?
This is the upper half of an ellipsoid.
(d) Sketch the graph of this function.