1. Set up, but do not evaluate, an integral for the volume bounded by the planes \( x + y + z = 4 \) and \( z = 0 \), and the parabolic cylinders \( y = 1 - x^2 \) and \( y = x^2 - 1 \).

Start by sketching the region in the \( xy \)-plane between the parabolas \( y = 1 - x^2 \) and \( y = x^2 - 1 \). We can use this sketch to find bounds on \( x \) and \( y \):

\[
x^2 - 1 \leq y \leq 1 - x^2 \\
-1 \leq x \leq 1
\]

To set up the integral, we now observe that the plane \( z = 4 - x - y \) is always above the plane \( z = 0 \) on this region. We can tell because the place where the two planes intersect is \( 0 = 4 - x - y \), or the line \( y = 4 - x \) in the \( xy \)-plane. Including this in the sketch in the \( xy \)-plane, we see that this does not touch the region over which we’re integrating. This is the volume integral:

\[
V = \int_{-1}^{1} \int_{x^2-1}^{1-x^2} (4 - x - y) \, dy \, dx
\]

2. Find the volume enclosed by the paraboloid

\[ z = x^2 - y^2 + 2 \]

and the planes \( x = 0 \), \( x = 3 \), \( y = -1 \), \( y = 1 \), and \( z = 0 \).

\[
V = \int_{-1}^{1} \int_{0}^{3} (x^2 - y^2 + 2) \, dx \, dy
\]

\[
= \int_{-1}^{1} [(1/3)x^3 - xy^2 + 2x]_{0}^{3} \, dy
\]

\[
= \int_{-1}^{1} (9 - 3y^2 + 6) \, dy
\]

\[
= \int_{-1}^{1} (15 - 3y^2) \, dy
\]

\[
= 15y - y^3|_{-1}^{1} = 28
\]
3. Evaluate the integral.

\[
\int_0^{\sqrt{\pi}} \int_{y/4}^{\sqrt{\pi}} \sin(x^2) \, dx \, dy
\]

We’ll switch the order of integration, since finding an antiderivative with respect to \( x \) looks problematic.

\[
\int_0^{\sqrt{\pi}} \int_{y/4}^{\sqrt{\pi}} \sin(x^2) \, dx \, dy = \int_0^{\sqrt{\pi}} \int_0^{4x} \sin(x^2) \, dy \, dx
\]

\[
= \int_0^{\sqrt{\pi}} y \sin(x^2) \bigg|_{y=4x}^{y=0} \, dx
\]

\[
= \int_0^{\sqrt{\pi}} 4x \sin(x^2) \, dx
\]

Do a substitution with \( u = x^2, \, du = 2x \, dx \).

\[
= \int_0^\pi 2 \sin u \, du
\]

\[
= -2 \cos u \bigg|_0^\pi = -2(-1 - 1) = 4
\]