Quiz 5: 15.4-15.6

Name

Math 2263
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1. Set up, but do not evaluate, the following integrals.

(a) A double integral in polar coordinates which gives the area inside the cardioid \( r = 1 + \sin \theta \) in the right half of the plane (where \( x \)-values are positive)

(b) A triple integral which gives the volume between the cone \( x = \sqrt{9y^2 + 4z^2} \) and the plane \( x = 6 \)
2. Suppose $x$ and $y$ are random variables with joint density function $f$.

$$f(x, y) = \begin{cases} 
3xy^2 & \text{if } 0 \leq x \leq 1, -1 \leq y \leq 1 \\
0 & \text{otherwise}
\end{cases}$$

Find the probability that $x \leq 1/2$ and $y \geq 0$.

3. Evaluate the integral by converting it to polar coordinates.

$$\int_{-2}^{0} \int_{0}^{\sqrt{4-y^2}} \cos(x^2 + y^2) \, dx \, dy$$