Integral Review Worksheet Solutions

Math 2263
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(a) \( \iint_S \mathbf{F} \cdot d\mathbf{S} \), where \( \mathbf{F} = xy \mathbf{i} - \frac{1}{2} y^2 \mathbf{j} + zk \) and \( S \) is the union of the three surfaces 
\( z = 4 - 3x^2 - 3y^2, 1 \leq z \leq 4 \), with upward-pointing normal; 
\( x^2 + y^2 = 1, 0 \leq z \leq 1 \) with outward-pointing normal; and \( z = 0 \), with downward-pointing normal

Apply Gauss’ Divergence Theorem. (vi)

(b) \( \iint_S xy \, dS \), where \( S \) is the portion of the plane \( x + y + z = 1 \) where 
\( x \geq 0, y \geq 0, z \geq 0 \)

Use the regular surface integral method. (vii)

(c) \( \iiint_W e^{\sqrt{x^2+y^2+z^2}} \, dV \), where \( W \) is the solid upper half of the unit sphere

Change variables to spherical coordinates. (iii)

(d) \( \int_C (4xyz, 2x^2z, -2x^2yz^2) \cdot d\mathbf{r} \), where \( C \) is the upper half of the ellipse \( x^2 + 9y^2 = 1 \) in the plane \( z = 1 \), traversed clockwise

Apply the Fundamental Theorem of Line Integrals. (ii)

(e) \( \iiint_V (-3x^2z - 3y^2z) \, dV \) where \( V \) is the interior of the cone \( z = \sqrt{x^2+y^2} \) with \( 0 \leq z \leq 1 \)

Change variables to cylindrical coordinates. (iv)

(f) \( \iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S} \), where \( \mathbf{F} = (-y - 1)\mathbf{i} + (x + \sin(xz^4))\mathbf{j} + (y^2 + e^{-z})\mathbf{k} \) and \( S \) is the union of the surfaces \( x^2 + y^2 = 1, 0 \leq z \leq 1 \), and \( x^2 + y^2 + (z - 1)^2 = 1, 1 \leq z \leq 2 \), both with inward-pointing normal

Apply Stokes’ Theorem. (v)

(g) \( \int y^3 \, dx - x^3 \, dy \) where \( c \) is the positively-oriented circle of radius 2 centered at \((0, 0)\)

Apply Green’s Theorem. (i)

(i) \( \int_0^{2\pi} \int_0^2 -3r^3 \, dr \, d\theta \) \hspace{2cm} (iii) \( \int_0^{\pi/2} \int_0^{2\pi} e^\rho \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi \)

(ii) \( f(1, 0, 1) - f(-1, 0, 1) \), where 
\( f(x, y, z) = \frac{2x^2y}{z} \)
(iv) $\int_0^1 \int_0^{2\pi} \int_0^z -3r^3z \, dr \, d\theta \, dz$

(v) $-\int_0^{2\pi} (\sin t + 1) \, dt$

(vi) $\int_0^{2\pi} \int_0^1 \int_0^{4-3r^2} r \, dz \, dr \, d\theta$

(vii) $\int_0^1 \int_0^{1-u} 6\sqrt{3}uv \, dv \, du$