16.4#8
Use Green’s Theorem to evaluate the line integral along the given positively oriented curve.

\[ \int_C xe^{-2x} \, dx + (x^4 + 2x^2y^2) \, dy \]

\( C \) is the boundary of the region enclosed by the parabolas \( y = x^2 \) and \( x = y^2 \). Green’s Theorem applies because \( C \) is the boundary of a closed region \( D \), which is most easily described in polar coordinates \( 1 \leq r \leq 2 \) and \( 0 \leq \theta \leq 2\pi \).

\[
\int_{\partial D} \mathbf{F} \cdot d\mathbf{r} = \iint_D \left[ \frac{\partial}{\partial x} (x^4 + 2x^2y^2) - \frac{\partial}{\partial y} (xe^{-2x}) \right] \, dA
\]

\[
= \iint_D \left[ 4x^3 + 4xy^2 \right] \, dA
\]

\[
= \iint_D \left[ 4x(x^2 + y^2) \right] \, dA
\]

\[
= \int_0^{2\pi} \int_1^2 4r \cos \theta (r^2) \, dr \, d\theta
\]

\[
= \int_1^2 \int_0^{2\pi} 4r^4 \cos \theta \, d\theta \, dr
\]

\[
= \int_1^2 -4r^4 \sin \theta \bigg|_0^{2\pi} \, dr
\]

\[
= 0
\]

16.4#18
A particle starts at the point \((-2, 0)\), moves along the \( x \)-axis to \((2, 0)\), then along the semicircle \( y = \sqrt{4 - x^2} \) to the starting point. Use Green’s Theorem to find the work done on this particle by the force field \( \mathbf{F}(x, y) = (x, x^3 + 3xy^2) \).

We may use Green’s Theorem since \( C \) is a closed curve. If \( D \) is the region inside that curve, we may most easily describe \( D \) in polar coordinates by \( 0 \leq \theta \leq \pi \) and \( 0 \leq r \leq 2 \).

\[
\int_C \langle x, x^3 + 3xy^2 \rangle \cdot d\mathbf{r} = \iint_D \left[ \frac{\partial}{\partial x} (x^3 + 3xy^2) - \frac{\partial}{\partial y} (x) \right] \, dA
\]

\[
= \iint_D \left[ (3x^2 + 3y^2) - 0 \right] \, dA
\]

\[
= \int_0^\pi \int_0^2 3r^2 r \, dr \, d\theta
\]
\[
= \int_0^\pi \int_0^2 3r^3 \, dr \, d\theta \\
= \int_0^\pi \left. \frac{3}{4} r^4 \right|_0^2 \, d\theta \\
= \int_0^\pi 12 \, d\theta \\
= 12\pi
\]