Use the Divergence Theorem to calculate the surface integral $\iint_{S} F \cdot dS$; that is, calculate the flux of $F$ across $S$, when

$$F(x, y, z) = x^3 yi - x^2 y^2 j - x^2 yz k,$$

and $S$ is the surface of the solid bounded by the hyperboloid $x^2 + y^2 + z^2 = 1$ and the planes $z = -2$ and $z = 2$.

$$\text{Div}(F) = 3x^2 y - 2x^2 y - x^2 y = 0$$

$$\iint_{S} F \cdot dS = \iiint_{E} \text{Div}(F) \, dV = \iiint_{E} 0 \, dV = 0$$

Use the Divergence Theorem to calculate the surface integral $\iint_{S} F \cdot dS$; that is, calculate the flux of $F$ across $S$, when

$$F(x, y, z) = x^2 yi + xy^2 j + 2xyz k,$$

and $S$ is the surface of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + 2y + z = 2$.

$$\text{Div}(F) = 2xy + 2xy + 2xy = 6xy$$

$$\iint_{S} F \cdot dS = \iiint_{E} \text{Div}(F) \, dV$$

$$= \int_{0}^{2} \int_{0}^{2-2y} \int_{0}^{2-x-2y} 6xy \, dz \, dx \, dy$$

$$= \frac{2}{5}$$