You may use a scientific calculator, but you may not use books, notes, graphing calculators, or your neighbors’ papers. Sign your name below to certify that you followed these instructions.

Signature:

Show all work for each problem so that partial credit may be given. Answers without supporting work will receive no credit. Clearly indicate your answers. If you need more space, use the blank sheet at the back of your test booklet.

Make sure that you have 5 problems on your test. If any exam material is missing, or if any questions are unclear, raise your hand or bring your exam to the proctor.

You may find the following facts useful:

\[
\sin^2 \alpha = \frac{1}{2} \left( 1 - \cos(2\alpha) \right)
\]

\[
\cos^2 \alpha = \frac{1}{2} \left( 1 + \cos(2\alpha) \right)
\]

Good luck!
1. Let
\[ \mathbf{F}(x, y, z) = \langle z^3 e^{yz} - 2xy, x^5 z^4 \cos(xz^3) + y^2, x^2 y^3 + z^2 \rangle. \]

Evaluate the surface integral
\[ \iint_S \mathbf{F} \cdot d\mathbf{S}, \]
where \( S \) is the surface consisting of the upper half of the sphere \( x^2 + y^2 + z^2 = 1 \) between \( z = 0 \) and \( z = 1 \), and the disk \( x^2 + y^2 \leq 1 \) in the plane \( z = 0 \), positively oriented.
2. Consider the vector field
\[ \mathbf{G}(x, y, z) = \left( (y + 1)x e^y - e^z \right) \mathbf{i} + \left( 2xyz - ye^y \right) \mathbf{j} + \left( -xz^2 \right) \mathbf{k}. \]

(a) Find \( \text{curl}(\mathbf{G}) \).

(b) Find \( \text{div}(\mathbf{G}) \).

(c) Could \( \mathbf{G} = \text{curl}(\mathbf{F}) \) for some field \( \mathbf{F} \)? Why or why not?

(d) Is \( \mathbf{G} \) conservative? Why or why not?
3. Let $C$ be the curve forming the boundary of the part of the surface $z = x^2 + 2xy$ which lies inside the cylinder $x^2 + y^2 = 4$, oriented downward. Compute the integral

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where $\mathbf{F}$ is the vector field

$$\mathbf{F}(x, y, z) = (2xz + y^3 + y + x^2\sqrt{x^5 + 1})\mathbf{i} + (3xy^2 + x)\mathbf{j} + (2y^2 + x^2)\mathbf{k}.$$
4. Find the flux of the vector field

\[ \mathbf{F}(x, y, z) = (2x, y + z, z - y) \]

across the surface of the cylinder \( y^2 + z^2 = 4 \) between the planes \( x = 0 \) and \( x = y + 3 \), oriented away from the \( x \)-axis.
5. Evaluate the integral
\[
\iint_S 4x \, dS,
\]
where \( S \) is the surface parametrized by \( x = 2s, y = 3t + s, z = s^2 + 1 \), with \( 0 \leq s \leq 1 \) and \( 1 \leq t \leq 2 \).