Math 2263 Multivariable Calculus  
Worksheet: Vectors and Matrices

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This worksheet is intended to give you practice performing basic computations with vectors and matrices.

1. Let \( \mathbf{a} = \langle 3, 4, -1 \rangle \) and \( \mathbf{b} = \langle -2, 4, -5 \rangle \). Compute each sum.
   (a) \( \mathbf{a} + 2\mathbf{b} \) 
   (b) \( 3\mathbf{a} - \frac{1}{2}\mathbf{b} \)

2. Let \( \mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^3 \). Which expressions make sense?
   (i) \((\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}\) 
   (ii) \(\mathbf{a}(\mathbf{b} \cdot \mathbf{c})\) 
   (iii) \(\mathbf{a} \cdot \mathbf{b} + \mathbf{c}\) 
   (iv) \(|\mathbf{a}|(\mathbf{b} \cdot \mathbf{c})\) 
   (v) \(\mathbf{a} \cdot |\mathbf{a} \cdot \mathbf{c}|\)

3. Find the unit vector in the direction of each vector.
   (a) \(\langle 2, -3, 1 \rangle\)
   (b) \(\frac{2}{3}\mathbf{i} + \mathbf{j} - \frac{1}{2}\mathbf{k}\)

4. Find \(\mathbf{a} \cdot \mathbf{b}\) for each pair of vectors \(\mathbf{a}\) and \(\mathbf{b}\).
   (a) \(\mathbf{a} = \langle -4, 5, 2 \rangle, \mathbf{b} = \langle -3, 4, -5 \rangle\)
   (b) \(\mathbf{a} = \langle \frac{1}{2}, -1 \rangle, \mathbf{b} = \langle \frac{3}{2}, 3 \rangle\)
   (c) \(\mathbf{a} = -\mathbf{i} - 2\mathbf{k}, \mathbf{b} = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}\)
   (d) \(\mathbf{a} = s\mathbf{i} + 2s\mathbf{j}, \mathbf{b} = -t\mathbf{i} + t^2\mathbf{j} + 3t\mathbf{k}\)
   (e) \(|\mathbf{a}| = 3, |\mathbf{b}| = 4\), the angle between \(\mathbf{a}\) and \(\mathbf{b}\) is 45°
5. Determine whether each pair of vectors is parallel, perpendicular, or neither.
   (a) \( \langle 4, 6 \rangle, \langle -3, 2 \rangle \)
   (b) \( \langle -5, 3, 7 \rangle, \langle 6, -8, 2 \rangle \)
   (c) \(-i + 2j + 5k, 3i + 4j - k\)
   (d) \(2i + 6j - 4k, -3i - 9j + 6k\)

6. Consider the points \( P(1, -3, -2), Q(2, 0, -4), R(6, -2, -5) \).
   (a) Find \(|a|, |b|, \) and \(|c|\), where \( a = \overrightarrow{PQ}, b = \overrightarrow{PR}, \) and \( c = \overrightarrow{QR}. \)
   (b) Find \( a \cdot b, a \cdot c, \) and \( b \cdot c. \)
   (c) Consider the triangle with \( P, Q, \) and \( R \) as vertices. Is this a right triangle? Give two arguments, one using the information from (a) and another using the information from (b).
7. Suppose \( A \) is a \( 3 \times 2 \) matrix, \( B \) is a \( 2 \times 3 \) matrix, \( C \) is a \( 2 \times 2 \) matrix, and \( D \) is a \( 1 \times 3 \) matrix. Which expressions make sense?

(i) \( AB \)  
(ii) \( A^T B \)  
(iii) \( DA \)  
(iv) \( AD \)  
(v) \( C A^T \)  
(vi) \( \det(A) \)  
(vii) \( \det(C) \)  
(viii) \( \det(AC) \)  
(ix) \( \det(BA) \)

8. Consider the matrices

\[ A = \begin{pmatrix} 1 & 3 \\ 3 & 3 \\ 4 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 4 & -1 \\ 5 & 3 & -3 \end{pmatrix}. \]

(a) Find \( AB \).

(b) Find \( BA \).

9. Find all possible values of \( a \) if

\[
\begin{vmatrix}
-2 & a & 1 \\
-3 & 4 & a \\
3 & -1 & 1
\end{vmatrix} = 7.
\]
10. Consider the matrices

\[ T = \begin{pmatrix} 2 & 4 \\ -3 & -1 \end{pmatrix}, \quad U = \begin{pmatrix} 3 & -5 \\ 2 & 1 \end{pmatrix}. \]

(a) Find \( \det(T) \).

(b) Find \( \det(U) \).

(c) Find \( \det(TU) \).

11. Consider the matrices

\[ M = \begin{pmatrix} -3 & 2 & 1 \\ 1 & 3 & 5 \\ -4 & -1 & -2 \end{pmatrix}, \quad N = \begin{pmatrix} -2 & 4 & -1 \\ -5 & 3 & -3 \\ 1 & 2 & 1 \end{pmatrix}. \]

(a) Find \( \det(M) \).

(b) Find \( \det(N) \).

(c) Find \( \det(MN) \).

12. Looking at your work from the previous two problems, observe that in both cases we have \( \det(A)\det(B) = \det(AB) \). Prove that this is always true for \( 2 \times 2 \) matrices. (In fact it is true in general! For extra fun, you might try proving the general case.)