1. There are two forms of line integrals we’ve addressed:

(I) \[ \int_{c} F \, ds = \int_{a}^{b} F(c(t)) \| c'(t) \| \, dt \]

(II) \[ \int_{c} F \cdot ds = \int_{a}^{b} F(c(t)) \cdot c'(t) \, dt \]

Decide whether to use approach (I) or (II) when you want to

(a) integrate the function \( g(x, y, z) = 3xy - y^2z^5 \) along the path \( c(t) = (t, 2, t - t^2) \) from the point \((-1, 2, 0)\) to \((0, 2, 0)\);

(b) integrate \( F(x, y, z) = (z, xy, 2y) \) along the path \( c(t) = (e^t, \ln t, te^t) \) from \( t = 0 \) to \( t = 2 \);

(c) integrate \( f(x, y) = (x^2y, y) \) along the path \( c(t) = (\cos t, \sin t) \) from \( t = 0 \) to \( t = 2\pi \);

(d) integrate \( F(x, y, z) = y \) along the path \( c(t) = (t^2, 1, 3t^3) \) from \((0, 1, 0)\) to \((1, 1, 3)\).

2. Kermit is hopping between two lily pads at \((0, 0, 0)\) and \((2, 4, 0)\). Gravity together with the wind over his swamp give a total force field of \((2xy, x^2, -2)\) through which he hops. The path Kermit plans to follow is given by

\[ c_1(t) = (t, 2t, 2t - t^2). \]

(a) Find how much work Kermit will do by integrating the force field along his path.

(b) There’s a tasty bug out of reach of Kermit’s intended path, so instead he jumps extra high to catch it. His new path is

\[ c_2(t) = (t, 2t, 4t - 2t^2). \]

If eating the bug will give Kermit 2 units of energy, is it worth it for him to follow the second path?

(c) Notice that the force field \( F(x, y, z) = (2xy, x^2, -2) \) is actually the gradient of a function \( f(x, y, z) \). Find that function \( f \).
(d) Now compute the work again using the Fundamental Theorem of Line Integrals, which says that

\[ \int_c \nabla f \cdot ds = f(c(b)) - f(c(a)), \]

where the path \( c(t) \) goes from \( t = a \) to \( t = b \). Use this to confirm your solutions in parts (a) and (b).

(e) Based on your experience in this problem, what should be the first step you take to compute the line integral of a vector field?

3. Godzilla has come to terrorize Tokyo. The giant lizard rises from Tokyo bay at time \( t = 0 \). His path of destruction is given as a function of time by \( x(t) = \sin(t) - t \cos(t) \) and \( y(t) = \cos(t) + t \sin(t) \) until he returns to the deep at time \( t = 2\pi \). If the cost to rebuild a section of Tokyo (in yen) is given as a function of the \( xy \) coordinates of the city \( C(x, y) = 100000x^2 + 150000y^2 \).

(a) If how far does Godzilla walk, if units are given in kilometers?
(b) How much will it cost to rebuild Tokyo?