Due date: Wednesday, July 24

Short Answer
State the answer to each of the following questions. It is not necessary to write down any justification (though of course one would want to be able to explain how each solution was obtained).

1. For $0 \leq i \leq 5$, let $x_i = (i^2 + i)/30$ and define $P = \{ x_i : i = 0, 1, \ldots, 5 \}$.
   
   (a) Show that $P$ is a partition of $[0, 1]$.
   (b) Find $\mu(P)$.
   (c) Find a refinement $P_0$ of $P$ such that $\mu(P_0) = 1/15$.

2. Give an example of a function $f : [a, b] \to \mathbb{R}$ where $f \in R(x)$ on $[a, b]$, but there is no antiderivative $F : [a, b] \to \mathbb{R}$ where $F' = f$.

Proofs
For the remaining problems, a proof is required. In particular, a complete solution must be stated in sentence form with appropriate justification for each step.

1. Suppose $f : [a, b] \to \mathbb{R}$ is decreasing. Prove that $f \in R(x)$ on $[a, b]$. That is, prove the case of Theorem 5.3 which is omitted from the proof in the text.

2. Prove Theorem 5.7. That is, suppose $f : [a, b] \to \mathbb{R}$ is bounded. Then $f \in R(x)$ on $[a, b]$ iff, for each sequence $\{P_n\}_{n=1}^\infty$ of marked partitions with $\{\mu(P_n)\}_{n=1}^\infty$ converging to zero, the sequence $\{S(P_n, f)\}_{n=1}^\infty$ is convergent.

3. (a) Suppose $f$ is integrable on $[-b, b]$ and $f$ is an odd function, that is, $f(-t) = -f(t)$ for all $t \in [-b, b]$. Prove that $\int_{-b}^{b} f \, dx = 0$.
   (b) Suppose $f$ is integrable on $[-b, b]$ and $f$ is an even function, that is, $f(-t) = f(t)$ for all $t \in [-b, b]$. Prove that $\int_{-b}^{b} f \, dx = 2 \int_{0}^{b} f \, dx$. 