1. Find the change of measure $dQ/dP$ that turns the following processes into martingales on the time interval $[0, T]$. To do that, write down $dX_t$, and change the definition of Brownian motion in a way that kills the $dt$ term.

   (a) $X_t = B_t - 2t$
   (b) $X_t = e^{B_t}$
   (c) $X_t = e^{B_t + t}$

2. Compute

   $$d\left((B_t + qt)^2 - t\right)(e^{-qB_t - \frac{1}{2}q^2t})$$

   and finish our proof from lecture that $\tilde{B}_t$ is a Brownian motion under the measure $Q$ given in Girsanov’s theorem.

3. Use the Black–Scholes pricing formula to compute the (current, time-0) price of a European call in a market with $S_0 = 100$, $r = 0.01$, strike $K = 100$, and maturity $T = 3.7$. Obtain a numeric answer.

   Recall the call–put parity: a put option has pay-off $(K - S_T)_+$ (it allows you to sell one unit of stock for $K$). Let $C_t(K, T)$ be the price of a call option with strike $K$ and maturity $T$ at time $t$, and let $P_t(K, T)$ be the same for a put option. Because $(S_T - K)_+ - (K - S_T)_+ = S_T - K$, we have $C_T(K, T) - P_T(K, T) = S_T - K$.

   Recall that under the measure $Q$, discounted portfolio prices are martingales. Take conditional expectations (don’t forget the discounting!) to derive the relationship between fair prices $C_t(K, T)$ and $P_t(K, T)$ at time $t \leq T$.

   Finally use the relationship you derived to obtain a numeric value of the put option with the same data as you had for the call option above there. Check that your answer makes sense (in particular, should the call or the put be more expensive?).