1. (15 pts) Let $B_t$ be a Brownian motion. Let $X = B_1 + B_2$, and let $Y = B_4 - B_3$.
   
   (a) Find $E[X^3]$.
   
   (b) Find $\text{Cov}(X,Y)$.
   
   (c) Write down the joint distribution of $X$ and $Y$. Expand any matrix products in your answer.

   **Solution:**
   
   (a) $X \sim N(0,4)$ has a distribution that’s symmetric around the origin, so $E[X^3] = 0$.
   
   (b) $X$ depends on the values of Brownian motion up to time 2, $Y$ depends on the values from time 3 forward, so $X$ and $Y$ are independent. $\text{Cov}(X,Y) = 0$.
   
   (c) 
   
   $f_{X,Y}(a,b) = f_X(a)f_Y(b) = \frac{1}{4\pi} e^{-\frac{a^2}{4} - \frac{b^2}{4}}$.

2. (12 pts) For each of the following statements, determine whether they are true or false. You do not need to justify your answer. Below, $B_t$ is a Brownian motion.
   
   (a) If $X = B_1$ and $Y = B_2$ then $X$ and $Y$ are independent. **F**
   
   (b) If $X = B_1$ and $Y = B_2$ then the distributions of $X$ and $Y$ are absolutely continuous with respect to each other (in both directions). **T**
   
   (c) If $X = B_1$ and $Y = B_2$ then $X^2 + Y$ is a normal random variable. **F**
   
   (d) If $X = B_1$ and $Y = B_2$ then $3X + 17Y$ is a normal random variable. **T**
   
   (e) If $X = B_1$ and $Y = B_2$ then $X$ is $\sigma(Y)$-measurable. **F**, it’s measurable with respect to the $\sigma$-field that looks at everything up to time 2, but not with respect to $B_2$ alone.
   
   (f) If $X = B_1$ and $Y = B_2$ then $X + Y$ and $X - Y$ are independent. **F**, their covariance is $\text{Cov}(X+Y, X-Y) = 1 - 4 = -3$. 

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