M1371: Selected Exercises

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The following is a subjective list of problems, theorems and bullet points to study for the course M1371, as taught on Fall 2017. Listed exercises are taken from the worksheet pages and the examples are taken from the textbook *Calculus for Engineering*, by Miracle, C. (second edition). The author warns the reader that, this is a subjective selection of problems, and by no means it is claimed to be complete nor sufficient to pass the examinations in this course. Exercises marked by an asterisk (*) are of particular interest.

1 Exam I: Functions and Limits

2 Exam II: Derivatives

3 Exam III: Integrals

3.1 Linearization
Given a function $f(x)$ and a point $x = c$, its linearization is given by

$$L(x) = f(c) + f'(c)(x - c)$$

Exercise list.

- 5801. 3*,4*
- 5802. 7,8

3.2 Newton’s Method
Given a function $f(x)$ and an initial guess equal to $c$, the following process approximates one of the roots of the function

$$\text{Next guess} = c - \frac{f(c)}{f'(c)}$$

Exercise list.

- 5803. 1*
- 5806. 2
3.3 Anti-derivatives

By Riemann Sums

\[ \int f(x) \, dx = \lim_{n \to \infty} b - a \sum_{k=1}^{n} f \left( a + \frac{2k - 1}{2n} (b - a) \right) \]

It is useful to have memorized the following expressions (C refers to a constant),

\[ \sum_{k=1}^{n} C = Cn, \quad \sum_{k=1}^{n} (2k - 1) = n^2, \quad \sum_{k=1}^{n} (2k - 1)^2 = \frac{n}{3}(4n^2 - 1). \]

Here, you should be comfortable in computing the Riemann sums for linear and quadratic functions; e.g. \( x^2 + x + 1 \).

Exercise list.

- 5811: 1, 2
- 5812: 4

By Formulas and Substitution

Refer to this page for a comprehensive list of elementary integrals. It is expected that you know the following:

- Basic properties of Integrals.
- Integrals of polynomials; \( x^n, x^{-1} \).
- Integrals of \( \sin x, \cos x, \sec^2 x, \csc^2 x, \sec x \tan x, \csc x \cot x \).
- Integrals of \( e^x, ax^2, \ln x \).
- The “reverse” inverse trigonometric integrals.

It is, in my opinion, unlikely that you will be asked elementary integrals, you should be practicing substitution (a.k.a. reverse chain rule, U-substitution) exercises. Practice is your only friend here.

Exercise list.

- 5805: 1-7
- 5812: 3
- 5826: [1-6]*

Definite integrals

Say that \( \int f(x) \, dx = F(x) \). Then, the definite integral \( \int_{a}^{b} f(x) \, dx \) is given by

\[ F(b) - F(a) \]

Exercise list.

- 5815: 1
3.4 Application of Anti-derivatives

Displacement
Given a velocity function, $v(x)$, the position, $p(x)$, can be obtained by the integral

$$p(x) = \int v(x) \, dx.$$

Exercise list.
• 5816: 1
• 5822: 3*

Area between the $x$-axis and a curve
Recall that area is a positive quantity, thus, if we denote the absolute value by vertical bars, the area of a curve from $a$ to $b$ is given by

$$\left| \int_{a}^{b} f(x) \, dx \right|.$$

Keep in mind that certain issues may arise. For example, if the function intersects the axis multiple times. If this is the case you must split the integral by the intersection points.

Exercise list.
• 5811: 3

Area between two curves
The area between two functions $f(x)$ and $g(x)$ is given by

$$\left| \int_{a}^{b} f(x) - g(x) \, dx \right|.$$

As in the previous subsection, special care must be taken when the functions intersect. If this happens, you must find the intersections and then split the integral accordingly.

Exercise list.
• 5813: 2
• 5821: 1*,2*

Volumes
Make a sketch of the region to be revolved; I expect that the following sketch is representative of any problem that might be given in the examination.
Then, if the region is to be *rotated along the x axis*, the volume is given by the disk method:

\[
\text{Volume} = \pi \int_a^b [f(x)]^2 - [g(x)]^2 \, dx.
\]

Else, if the region is to be *rotated along the y-axis*, the volume is given by the shell method:

\[
\text{Volume} = 2\pi \int_a^b x[f(x) - g(x)] \, dx.
\]

**Exercise list.**
- **5814**: 1*,3* (Disk method)
- **5822**: 1*,2* (Shell method)

**Work**

Given a force function \( F(x) \), the work done over the domain \([a, b]\) is given by

\[
\text{Work} = \int_a^b F(x) \, dx.
\]

Some useful force functions are:

\[
F = \text{mass} \cdot \text{acceleration}, \quad F_s = kx.
\]

The latter is known as Hooke’s law.

**Exercise list.**
- **5823**: 1*,4*,5*
- **5833**: 3*,4

### 3.5 Mean Value Theorems

**For Derivatives**

If a function \( f(x) \), on an interval \([a, b]\), satisfies the following

i. The function \( f(x) \) is continuous on the *closed* interval \([a, b]\), and

ii. Its derivative \( f'(x) \) is defined on the *open* interval \((a, b)\)

then, we can conclude that there exist \( c \) in \([a, b]\) such that

\[
f'(c) = \frac{f(b) - f(a)}{b - a}
\]
Exercise list.

• 5812: 1*,2

For Integrals

If a function \( f(x) \), on an interval \([a, b]\) satisfies the following:

1. The function \( f(x) \) is continuous in the closed interval \([a, b]\)

then, we conclude that there exist \( c \) in \([a, b]\) such that

\[
f(c) = \frac{1}{b - a} \int_a^b f(x) \, dx
\]

Exercise list.

• 5825: 2*,3*

4 Miscellaneous

It wouldn’t harm you to know the Basic Theorem from page 354. Exercise 3 on 5824 might be relevant.