ARITHMETIC WITH FRACTIONS, MIXED NUMBERS, AND PRIME NUMBERS

As of October 24, 2005

The problems below are relevant for Grades 3-7, or possibly 2-7. According to the 2003 Minnesota Mathematics Standards, it is the goal of Minnesota that students become fluent in the arithmetic of fractions and mixed numbers by April of Grade 7. (which is the time of the Grade-7 state test) The problems below are intended to be done without a calculator. Of course, the whole-number arithmetic involved should be in line with the degree of complexity the Standards assign to whole-number arithmetic without a calculator. This being said, it is also important that some moderately large whole numbers appear in numerators or denominators so that students come to understand that there are some very general principles underlying specific calculations, especially as a preparation for algebra since in algebra the general principles are often the focus.

Notice that I have said these problems are for Grades 3-7, or possibly 2-7, even though, according to the Standards, it is only in Grades 5-7 that arithmetic with fractions and mixed numbers is regarded as relevant for the state tests. There is no inconsistency here. For state test readiness, which is the focus of the Standards, it is important that students develop skills over several years, especially in such a vast subject as arithmetic with fractions.

Prime numbers and least common multiples play a central role in adding and subtracting fractions and greatest common divisors are relevant for reducing fractions to lowest terms. Some problems involving prime numbers, least common multiples, and greatest common divisors are included below.

This link is focused on the arithmetic of fractions and mixed numbers. It might be that a future link will treat this arithmetic in conjunction with problems in context, but such problems are not included here.

Also, this link does not directly treat the meaning of fractions. My impression is that materials adopted for consistency with the pre-2003 standards give this topic appropriate emphasis.

Teachers and others should feel free to download this list and possibly cut off a bottom portion of the list in order to avoid showing younger children problems far beyond what they can do and/or cut off a top portion of the list to avoid giving the impression to older children that arithmetic with fractions is trivial.

I use the term ‘mixed number’ even though ‘mixed numeral’ would be more accurate; and I speak of ‘proper’ and ‘improper’ fractions, even though
here these adjectives have meanings that are unrelated to the meanings they have in ordinary discourse. Also, when fractions are studied, the phrase ‘is equivalent to’ is often used synonymously with ‘is equal to’ and ‘equals’—and similarly for the variations which are used as adjectives.

This list of problems is focused on certain skills. It does not treat meanings of fractions or reasons why certain operations work the way they do; these are also important, but they are not treated in this list of problems. However, my view is that, as a side benefit, honing of skills often increases understanding.

I am not involved in the construction of the state math tests, known as MCA’s, for 2006. Thus, I cannot speak about the extent, if any, to which skill on the problems below will be useful preparation for the 2006 MCA’s. However, I have kept the 2003 State Mathematics Standards in mind as I have written these exercises.

1. Calculate the sums:

\[
\frac{1}{3} + \frac{1}{3} = \frac{1}{5} + \frac{2}{5} = \frac{1}{6} + \frac{4}{6} = \\
\frac{1}{9} + \frac{1}{9} = \frac{3}{10} + \frac{4}{10} = \frac{2}{10} + \frac{5}{10} = \\
\frac{2}{7} + \frac{4}{7} = \frac{4}{37} + \frac{5}{37} = \frac{8}{13} + \frac{4}{13} = 
\]

2. Calculate the following differences:

\[
\frac{2}{3} - \frac{1}{3} = \frac{3}{5} - \frac{2}{5} = \frac{5}{6} - \frac{4}{6} = \\
\frac{7}{9} - \frac{5}{9} = \frac{9}{10} - \frac{6}{10} = \frac{8}{10} - \frac{5}{10} = \\
\frac{5}{7} - \frac{2}{7} = \frac{8}{12} - \frac{3}{12} = \frac{20}{29} - \frac{10}{29} = 
\]
3. Write each whole number as a fraction with denominator 3:

\[
3 = \quad 8 = \quad 1 = \\
10 = \quad 200 = \quad 15 = 
\]

4. Write each of the following mixed numbers as an improper fraction with denominator 4:

\[
1 \frac{3}{4} = \quad 3 \frac{1}{4} = \quad 5 \frac{2}{4} = \\
5 \frac{1}{2} = \quad 10 \frac{3}{4} = \quad 20 \frac{1}{4} = 
\]

5. Reduce the following fractions to lowest terms (meaning write an equivalent fraction that has as small a denominator as possible):

\[
\frac{3}{6} = \quad \frac{3}{9} = \quad \frac{20}{40} = \\
\frac{9}{15} = \quad \frac{10}{15} = \quad \frac{8}{10} = \\
\frac{90}{120} = \quad \frac{15}{35} = \quad \frac{2}{400} = 
\]
6. Find equivalent fractions having denominator 12 for each of the following fractions:

\[
\begin{align*}
\frac{1}{6} &= \frac{3}{4} = \frac{2}{3} \\
\frac{1}{2} &= \frac{2}{8} = \frac{6}{8} \\
\frac{5}{6} &= \frac{12}{15} = \frac{5}{20}
\end{align*}
\]

7. Reduce the following improper fractions to lowest terms (meaning write an equivalent improper fraction that has as small a denominator as possible):

\[
\begin{align*}
\frac{9}{6} &= \frac{36}{9} = \frac{100}{40} = \\
\frac{190}{120} &= \frac{12}{10} = \frac{30}{4} = \\
\frac{35}{15} &= \frac{350}{35} = \frac{38}{10} = 
\end{align*}
\]

8. Find equivalent improper fractions having denominator 8 for each of the following improper fractions:

\[
\begin{align*}
\frac{5}{4} &= \frac{11}{2} = \frac{18}{12} = \\
\frac{15}{6} &= \frac{20}{16} = \frac{18}{16} = \\
\frac{16}{4} &= \frac{8}{2} = \frac{36}{9} = 
\end{align*}
\]
9. Calculate each of the following sums and write the answer as a fraction (possibly improper) in lowest terms (that is with the smallest possible denominator which might equal 1 sometimes):

\[
\frac{3}{6} + \frac{1}{6} = \quad \frac{1}{12} + \frac{7}{12} =
\]

\[
\frac{7}{10} + \frac{9}{10} = \quad \frac{5}{6} + \frac{7}{6} =
\]

\[
\frac{18}{10} + \frac{17}{10} = \quad \frac{7}{8} + \frac{5}{8} =
\]

10. Calculate each of the following differences and write the answer as a fraction (possibly improper) in lowest terms (that is with the smallest possible denominator which might equal 1 sometimes):

\[
\frac{3}{6} - \frac{1}{6} = \quad \frac{27}{10} - \frac{3}{10} =
\]

\[
\frac{13}{12} - \frac{5}{12} = \quad \frac{55}{6} - \frac{7}{6} =
\]

\[
\frac{23}{14} - \frac{7}{14} = \quad \frac{131}{100} - \frac{96}{100} =
\]

5
11. Decide whether each of the following numbers is prime or composite. If it is composite write it as the product of prime numbers.

6 ________________ 11 ________________

12 ________________ 19 ________________

25 ________________ 30 ________________

36 ________________ 40 ________________

12. Calculate the following products:

\[
\frac{1}{2} \times \frac{3}{4} = \quad \frac{1}{3} \times \frac{1}{4} = \quad \frac{2}{3} \times \frac{3}{4} = \\
\frac{1}{3} \times \frac{3}{4} = \quad \frac{4}{3} \times \frac{5}{9} = \quad \frac{3}{10} \times \frac{7}{10} = \\
\frac{5}{2} \times \frac{3}{2} = \quad \frac{2}{9} \times \frac{2}{9} = \quad \frac{10}{3} \times \frac{3}{5} =
\]
13. Decide which answers for the preceding problem are in lowest terms. For those that are not, reduce them to lowest terms, keeping in mind that a whole number can be written as a fraction in lowest terms by using a denominator of 1.

14. Calculate the following quotients:

\[
\begin{align*}
\frac{1}{2} \div \frac{3}{4} &= \frac{1}{3} \div \frac{1}{4} = \frac{2}{5} \div \frac{2}{3} = \\
\frac{2}{5} \div \frac{3}{5} &= \frac{8}{7} \div \frac{5}{9} = \frac{3}{10} \div \frac{7}{10} = \\
\frac{5}{2} \div \frac{3}{2} &= \frac{7}{11} \div \frac{7}{11} = \frac{10}{7} \div \frac{3}{10} =
\end{align*}
\]
15. Decide which answers for the preceding problem are in lowest terms. For those that are not, reduce them to lowest terms.

16. Check each answer in Problems 14 and 15 by doing an appropriate multiplication.
17. Calculate the following sums:

\[
\frac{1}{4} + \frac{1}{2} = \frac{5}{12} + \frac{1}{6} = \frac{5}{8} + \frac{3}{4} =
\]

\[
\frac{3}{4} + \frac{5}{32} = \frac{3}{14} + \frac{2}{7} = \frac{7}{10} + \frac{18}{50} =
\]

\[
\frac{5}{9} + \frac{4}{27} = \frac{19}{16} + \frac{7}{4} = \frac{11}{14} + \frac{1}{2} =
\]

18. Decide which answers in the preceding problem are in lowest terms. For those that are not, reduce them to lowest terms. Then for all nine answers decide if they are proper or improper fractions. For those that are improper rewrite them as mixed numbers.
19. Calculate the following differences:

\[
\begin{align*}
\frac{5}{4} - \frac{1}{2} &= \frac{5}{12} - \frac{1}{6} = \frac{5}{8} - \frac{1}{4} = \\
\frac{3}{4} - \frac{7}{32} &= \frac{11}{14} - \frac{2}{7} = \frac{7}{10} - \frac{11}{50} = \\
\frac{7}{9} - \frac{11}{27} &= \frac{49}{16} - \frac{7}{4} = \frac{11}{21} - \frac{1}{3} =
\end{align*}
\]

20. Decide which answers in the preceding problem are in lowest terms. For those that are not, reduce them to lowest terms. Then for all nine answers decide if they are proper or improper fractions. For those that are improper rewrite them as mixed numbers.
21. Perform the following calculations writing answers as fractions, proper or improper (lowest terms not required):

\[
\left( \frac{3}{8} + \frac{1}{4} \right) + \frac{5}{8} = \quad \frac{3}{8} + \left( \frac{1}{4} + \frac{5}{8} \right) =
\]

\[
\left( \frac{3}{5} - \frac{1}{5} \right) + \frac{3}{10} = \quad \frac{3}{5} - \left( \frac{1}{5} + \frac{3}{10} \right) =
\]

\[
\left( \frac{17}{12} + \frac{5}{4} \right) \times \frac{1}{6} = \quad \frac{17}{12} + \left( \frac{5}{4} \times \frac{1}{6} \right) =
\]
21 continued.

\[
\left(\frac{3}{5} \times \frac{7}{6}\right) \times \frac{8}{21} = \quad \frac{3}{5} \times \left(\frac{7}{6} \times \frac{8}{21}\right) = 
\]

\[
\left(\frac{16}{9} \div \frac{4}{11}\right) \times \frac{2}{33} = \quad \frac{16}{9} \div \left(\frac{4}{11} \times \frac{2}{33}\right) = 
\]

\[
\left(\frac{13}{20} \div \frac{6}{5}\right) - \frac{11}{40} = \quad \frac{9}{10} \div \left(\frac{4}{7} \div \frac{15}{28}\right) = 
\]

12
22. Find equivalent mixed numbers for the following improper fractions:

\[
\begin{align*}
27/2 &= \quad 23/5 &= \quad 16/7 = \\
70/13 &= \quad 578/5 &= \quad 753/9 = \\
52/17 &= \quad 455/3 &= \quad 5344/10 =
\end{align*}
\]
23. List all the divisors of each of the following whole numbers in increasing order, starting with 1 and ending with the given whole number:

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<thead>
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<tbody>
<tr>
<td>6</td>
<td>11</td>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>25</td>
<td>30</td>
<td>36</td>
<td>60</td>
</tr>
</tbody>
</table>

24. Use the answers to the preceding problem to calculate the following greatest common divisors, denoted by gcd:

\[
gcd(6, 20) = \quad gcd(25, 20) = \]

\[
gcd(6, 25) = \quad gcd(12, 20) = \]

\[
gcd(36, 60) = \quad gcd(30, 20) = \]

\[
gcd(36, 30) = \quad gcd(25, 30) = \]

\[
gcd(6, 60) = \quad gcd(6, 11) = \]
25. Use the answers to the preceding problem to calculate the following least common multiples, denoted by lcm:

\[
\begin{align*}
lcm(6, 20) &=  \quad lcm(25, 20) = \\
lcm(6, 25) &=  \quad lcm(12, 20) = \\
lcm(36, 60) &=  \quad lcm(30, 20) = \\
lcm(36, 30) &=  \quad lcm(25, 30) = \\
lcm(6, 60) &=  \quad lcm(6, 11) = \\
\end{align*}
\]
26. Use answers to the preceding problem in order to make the following calculations:

\[
\frac{1}{6} + \frac{7}{20} = 
\]

\[
\frac{25}{30} + \frac{7}{20} = 
\]

\[
\frac{53}{60} + \frac{5}{6} = 
\]

\[
\frac{12}{25} - \frac{7}{20} = 
\]

\[
\frac{7}{36} + \frac{7}{60} = 
\]

\[
\frac{11}{30} - \frac{11}{36} = 
\]

\[
\frac{1}{6} + \frac{1}{25} = 
\]

\[
\frac{1}{12} - \frac{1}{20} = 
\]

\[
\frac{6}{25} - \frac{1}{30} = 
\]

\[
\frac{11}{6} + \frac{6}{11} = 
\]

16
27. Decide which of the answers in the preceding problem are in lowest terms. For those that are not, reduce them to lowest terms. Then find equivalent mixed numbers for all the answers that are improper fractions.

28. Write the following whole numbers as products of primes. [If the whole number is itself a prime, we say that it, itself, is the ‘product’ even though no actual product is involved.]

   
   \[
   \begin{align*}
   6 &= 4 &= 10 &= 12 = \\
   9 &= 8 &= 15 &= 30 = \\
   20 &= 13 &= 72 &= 1600 = \\
   768 &= 113 &= 1001 &= 3125 = \\
   \end{align*}
   \]
29. Use the preceding problem to calculate the following greatest common divisors and least common multiples:

\[
gcd(4, 6) = \quad lcm(4, 6) =
\]

\[
gcd(8, 20) = \quad lcm(8, 20) =
\]

\[
gcd(768, 15) = \quad lcm(768, 15) =
\]

\[
gcd(4, 13) = \quad lcm(4, 13) =
\]

\[
gcd(1001, 13) = \quad lcm(1001, 13) =
\]

\[
gcd(72, 1600) = \quad lcm(72, 1600) =
\]
30. Use the preceding problem to decide which of the following fractions is in lowest terms, and to then reduce the ones that are not to lowest terms:

\[
\frac{4}{6}, \quad \frac{20}{8}, \quad \frac{15}{768}
\]

\[
\frac{4}{13}, \quad \frac{13}{1001}, \quad \frac{1600}{72}
\]

31. Use answers for Problem 29 to make the following calculations:

\[
\frac{15}{4} - \frac{11}{6} = \frac{5}{8} + \frac{3}{20} = \frac{8}{15} + \frac{313}{768} =
\]

\[
\frac{3}{4} - \frac{5}{13} = \frac{1}{13} - \frac{91}{1001} = \frac{147}{1600} + \frac{135}{768} =
\]
32. Use answers for Problem 28 to do the following multiplication and division problems using a much early cancellation as possible so that then the answers automatically are in lowest terms

\[
\frac{8}{15} \times \frac{13}{36} = \quad \frac{4}{9} \div \frac{13}{72} = \\
\frac{6}{1001} \times \frac{13}{8} = \quad \frac{9}{10} \div \frac{7}{30} = \\
\frac{10}{13} \times \frac{13}{72} = \quad \frac{30}{13} \div \frac{9}{1001} = \\
\frac{8}{7} \times \frac{11}{30} = \quad \frac{15}{20} \div \frac{5}{20} = \\
\frac{9}{3125} \times \frac{1600}{3} = \quad \frac{1}{8} \div \frac{1}{768} = 
\]
33. Check the answers to the multiplication exercises in the preceding problem by doing an appropriate division and check the answer to the division exercises by doing an appropriate multiplication.
34. By doing appropriate subtractions, list the following fractions in order, from smallest to largest:

\[
\frac{3}{5}, \frac{2}{3}, \frac{4}{7}, \frac{7}{12}, \frac{11}{20}, \frac{13}{20}, \frac{5}{8}, \frac{12}{19}, \frac{9}{14}, \frac{29}{48}
\]
35. Convert the mixed numbers to improper fractions:

\[
\begin{align*}
34 \frac{1}{2} & \quad 15 \frac{3}{5} & \quad 21 \frac{5}{8} \\
8 \frac{21}{25} & \quad 23 \frac{13}{100} & \quad 21 \frac{5}{8} \\
17 \frac{5}{13} & \quad 13 \frac{5}{17} & \quad 1000 \frac{1}{8}
\end{align*}
\]
36. Convert the improper fractions to mixed numbers

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<thead>
<tr>
<th>Fraction</th>
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<tbody>
<tr>
<td>$\frac{7}{2}$</td>
<td>$\frac{70}{21}$</td>
<td>$\frac{70}{19}$</td>
</tr>
<tr>
<td>$\frac{13}{8}$</td>
<td>$\frac{133}{11}$</td>
<td>$\frac{76}{16}$</td>
</tr>
<tr>
<td>$\frac{976}{11}$</td>
<td>$\frac{976}{22}$</td>
<td>$\frac{922}{23}$</td>
</tr>
</tbody>
</table>
37. Do the following calculations, writing the answers as mixed numbers with fractional parts in lowest terms. For this problem a proper fraction should be viewed as a mixed number with zero whole number part and a whole number should be viewed as a mixed number with zero fractional part.

\[
3 \dfrac{1}{8} \times 7 \dfrac{2}{5}
\]

\[
3 \dfrac{1}{8} + 7 \dfrac{2}{5}
\]

\[
4 \dfrac{2}{5} \div 2 \dfrac{4}{9}
\]
32 \frac{2}{5} - 18 \frac{4}{5}

\frac{3}{7} \times \left(4 \frac{5}{11} - 2 \frac{4}{5}\right)

\frac{2}{12} \div 7 \frac{3}{16}

The views encompassed by the preceding list of problems are mine alone. They should not be taken to be accurate or even approximate representations of positions of the State of Minnesota Department of Education. I did not consult with that Department while producing this list of problems.

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