The purpose of this link from my web-site is to identify a selection of problems aligned with the Minnesota mathematics standards and benchmarks for Grade 6 as adopted in Spring 2003. My focus consists of the standards and benchmarks themselves; the problems here serve to illuminate them. The benchmarks and standards that are particularly relevant for a particular problem are identified in the left-hand margins; for instance, 6-II.B.1 indicates the Grade-6 benchmark II.B.1 and 6-II.B refers to the corresponding standard. In another sense, the focus is the suitability of problems for the Minnesota Comprehensive Assessments (known as MCA’s), but in saying this I want to emphasize that the opinions are mine alone, formed without consultation with Minnesota Department of Education. This particular link also includes a variety of comments about the problems. A problem list without this commentary is on another link.

I was one of approximately 40 members of the mathematics subcommittee of the Academic Standards Committee, formed by the Minnesota Commissioner of Education in February 2003. I strongly support the mathematics standards and benchmarks resulting from the work of that committee and which, on the basis of a law passed by the Legislature and signed by the Governor, became official in Spring 2003. Although there is no guarantee that this web-site item reflects the thinking within the Department of Education, I have tried very hard to reflect the standards and benchmarks accurately, taking care not to bend them in the direction of my individual views. [Even though I strongly support the standards and benchmarks document, there are places where I would have preferred the document to be a bit different, and I suspect that the same is true (but not for the same places) of every member of the mathematics subcommittee.]

Anticipating that I might want to modify this document from time to time, I have refrained from labeling the problems with numerals and am planning to change the date at the top any time I make additions or changes. Since the standards are cumulative, all the K-6 benchmarks are relevant for the Grade-6 MCA. It seems to me that it is desirable for Grade-6 teachers to examine all the K-6 benchmarks giving special attention to those for grades 4-6, and in general for teachers to read the standards for a couple grades on either side of the grade they are teaching.

Even though I view all the problems below as consistent with the Grade-6 standards and benchmarks, the range of difficulty represented by them is wide. I have chosen the adjectives ‘standard’, ‘substantial’ and ‘challenging’ for the problems. The challenging problems are those that, in my opinion require sig-
significantly more than mastery of individual benchmarks. A substantial problem is one that has a feature of richness beyond what would be expected in a standard problem and yet which comes short of being the challenge that a challenging problem would represent. Among the standard problems I use two labels: standard-e, and standard-h indicating a distinction between easier and harder standard problems. This assignment of level of difficulty will follow each problem. But these personal opinions of mine are of secondary importance compared to the central issue of alignment of problems with benchmarks. I want to emphasize that the challenging and substantial problems are aligned with the benchmarks; it is not that they are on topics that go beyond the standards. [For an analogy, I mention a long-standing phenomenon with some standardized tests—a seventh grader might be told that he or she has performed at, say, the tenth-grade level. This does not mean that the seventh grader knows tenth-grade mathematics, but rather that he or she does as well on seventh-grade material as would an average tenth grader.]

Among the problems I include below are some which on first reading might seem appropriate for an MCA, but which nevertheless would not be. I point out, for instance, how one problem might have an unintentional cultural bias and thus would be bad for an MCA although possibly very good for classroom discussion.

The variety of different problems that are consistent with the standards and benchmarks is very large—that is the power of mathematics; a manageable number of basic principles and techniques enables one to handle a myriad of different situations. So, of course, the problem list that follows cannot be viewed as comprehensive.

For problems in which students are to place the correct digits in boxes, a decimal point or comma is included between appropriate pairs of boxes when relevant. If the answer requires fewer digits than boxes, it is the left-hand box or boxes which should be left blank. [If the Grade-6 MCA were, in fact, to include such problems it would be important that students become familiar with the instructions some days in advance of the test.]

There is not a sharp demarcation separating problems appropriate for various grade levels. For instance, some of the problems described below for Grade 6 are also in the link for Grade 5. Typically, a problem that is appropriate for both the Grade-6 MCA and the Grade-5 MCA would be regarded as a more difficult problem for a fifth grader than it would be regarded for sixth graders.

It is clear from the benchmarks 6-II.B.9, 6-II.B.10, and 6-II.B.11 that the Grade-6 MCA should contain a significant section where a calculator is permitted. But it is also clear from other benchmarks under standard 6-II.B that there should be a wide variety of problems which the student should be able to handle by hand; for instance, the adjective ‘common’ and the phrase involving the common denominator in benchmark 6-II.B.7 are clear indications that this benchmark concerns skill without a calculator.
FOR GRADE 6, WITH COMMENTS

The first part of the list below is relevant for the non-calculator portion of the Grade-6 MCA, and later, an introductory sentence identifies the place where the ‘calculator permitted’ portion begins.

I want to again emphasize: Although the standards and the benchmarks accompanying them constitute an official document of the state of Minnesota, all the judgments about alignment of problems with the benchmarks and standards are mine; neither do they have any official standing nor have they been obtained in consultation with the Minnesota Department of Education. Also, they have not been reviewed by the University of Minnesota where I am a faculty member and, of course, they do not represent any official view of that institution.

5-II.A.3 Which of the following fractions equals 75%?
(a) \( \frac{15}{20} \)
(b) \( \frac{16}{20} \)
(c) \( \frac{7}{5} \)
(d) \( \frac{7}{7} \)

Difficulty: standard-e.

5-II.A.1 Which of the following numerals represents forty-five and twenty-three thousandths?
(a) 0.4523
(b) 45.0023
(c) 45.023
(d) 45.23

Difficulty: standard-e. Notice that 0.068 is not listed as an option. Some might claim that a legitimate interpretation of the problem is that ‘forty-five’ and ‘twenty-three’ are both adjectives modifying ‘thousandths’, and that the conjunction ‘and’ denotes addition. Such a person should, however, be aware of the other possible interpretation that 45 is the whole number part of a decimal numeral, and then use this interpretation since 0.068 is not a given option. [An irregularity in names of numerals that did not arise in this problem, but of which all teachers should be aware, especially those who teach students for whom English is a second language: one billion in American English means \( 10^9 \)—that is, one thousand millions; whereas in British English it means \( 10^{12} \), which is one million millions. And of course, a similar issue arises in connection with one trillion.]
FOR GRADE 6, WITH COMMENTS

6-V.A.1 The following two-dimensional figure can be folded along the boundaries of the squares to give a cube. The point A becomes one of the vertices of the cube. Which other points become the same vertex of the cube that A becomes?

(a) B and C  
(b) B and D  
(c) only C  
(d) only D

Difficulty: standard-e.

5-V.C.3 Which is correct?

(a) ‘Acres’ and ‘square feet’ are both appropriate units for area.
(b) ‘Acres’ is an appropriate unit for length and ‘square feet’ is appropriate for area.
(c) ‘Acres’ is an appropriate unit for area and ‘square feet’ is appropriate for volume.
(d) ‘Acres’ is an appropriate unit for time and ‘square feet’ is appropriate for area.

Difficulty: standard-e. The difficulty is mostly in the reading of the problem. There might be a concern that this problem favors rural students who have used the term ‘acres’ more in their lives than have students in cities. I think the problem is ok, since the term ‘acre’ is a mathematical term.
6-II.B.1 How many 3’s are there in the prime factorization of 810?
(a) 0
(b) 1
(c) 2
(d) 4

Difficulty: standard-e. My judgment is that the level of difficulty would rise to standard-h or substantial according as 810 in the problem were replaced by 567 or 2349, respectively—it being harder in these cases to find the prime factors different from 3 (even though one can do the problem without being concerned about other prime factors). This problem could also be a non-multiple-choice problem in order to keep students from first trying to see if $3 \times 3 \times 3 \times 3 = 81$ is a divisor and if that doesn’t work trying $3 \times 3 = 9$, and so forth. I myself am ok with this approach to doing the problem—it shows somewhat the same understanding as does the approach of successively dividing by 3.

6-II.B.6 What does $1,000,000 \times 0.01$ equal?
(a) 1,000
(b) 10,000
(c) 100,000
(d) 100,000,000

Difficulty: standard-e

6-II.B.8 What does 5% of 2% equal?
6-II.B.7 (a) 0.1%
5-II.B.4 (b) 0.4%
5-II.A.3 (c) 10%
(d) 40%

Difficulty: standard-h.
6-II.B.7 \( \frac{\frac{2}{3}}{\frac{4}{9}} = \)

(a) \( \frac{8}{27} \)
(b) \( \frac{2}{3} \)
(c) \( \frac{3}{2} \)
(d) \( \frac{27}{8} \)

Difficulty: standard-h. This problem illustrates an advantage of some multiple-choice problems. The focus here is on being careful about what belongs in numerators and what belongs in denominators; it is clear that there will be no partial credit if one gets a fraction turned upside-down, while it gives a student a chance to correct an arithmetic error such as \( 9 \times 3 = 24 \). The vertical and slanting methods of writing fractions are both used extensively, so it is important that students be familiar with both.

6-III.A.1 In the rectangular coordinate system, as usually drawn, which of the following is true about the line through the points \((1, -2)\) and \((-1, -2)\):

(a) The line is horizontal.
(b) The line is vertical.
(c) The line is slanted with the lower part to the left and the higher part to the right.
(d) The line is slanted with the higher part to the left and the lower part to the right.

Difficulty: standard-h.

6-V.A.2 Denote the vertices of a triangular pyramid, all whose faces are equilateral triangles, by \( A, B, C, \) and \( D \). It is reflected in a plane containing the edge \( AB \) and passing through the midpoint of edge \( CD \). Which vertices move when this is done?

(a) none
(b) \( A \) and \( B \) only
(c) \( C \) and \( D \) only
(d) all four vertices

Difficulty: standard-h.
6-V.A.3  How many reflective symmetries does a regular right hexagonal prism have?
5-V.A.2  (Recall that a hexagon is a figure having six edges and six vertices and that the
4-V.B.1  lateral edges in a right prism are perpendicular to the base, which is a hexagon
in this case.)

(a) 3
(b) 4
(c) 6
(d) 7

Difficulty: substantial.

6-IV.A.2  Three positive numbers have the property that one of them is larger than the
6-I.2  sum of the other two. Which of the following four assertions is accurate.

(a) The mean of the three numbers will definitely be less than the median
(b) The mean of the three numbers will definitely equal the median of the
(c) The mean of the three numbers will definitely be larger than the median
(d) Neither (a) nor (b) nor (c) is an accurate statement.

Difficulty: challenging. This problem is quite easy provided the student adopts
an appropriate approach, but at the Grade-6 level finding that approach is
a challenging task. The correct approach is to consider two very different
examples—one, for instance, in which the three numbers are 1, 4, and 6 and
another in which they are 1, 4, and 100. Then simple calculations show that
(d) is the correct response.

5-II.B.4  Calculate
5-II.A.4

\[
\begin{array}{c}
37 \\
\times .79
\end{array}
\]

and round your answer to the nearest whole number.

Difficulty: standard-e.
6-II.B.7 Calculate \[
\frac{3}{7} - \frac{1}{35},
\]
writing your answer in lowest terms. Enter the numerator in the left-hand boxes and the denominator in the right-hand boxes.

Difficulty: standard-e. Partial credit for 14/35 seems appropriate to me.

6-II.B Calculate \((-9 \times 2) - 2\).

[There are enough boxes for one of them to be used for a negative sign if that is needed.]

Difficulty: standard-e.

6-I.3 How many minutes is it from 11:48am to 8:27pm of the next day?

Difficulty: standard-h.

6-II.B.7 The perimeter of a certain square is \(7\frac{1}{5}\) inches. Find the length of each side in inches, writing your answer as an improper fraction in lowest terms. Enter the numerator in the left-hand boxes and the denominator in the right-hand boxes.

Difficulty: standard-h. To me it seems reasonable to award partial credit for either of the answers 36/20 or 18/10, both of which would be correct except for the fact that these fractions are not in lowest terms.
For any year that is not a leap-year, find the one date that is in the center of that year—that is, find the date that has the same number of dates preceding it as following it. Enter the month in the left-hand array of boxes and the appropriate date within that month in the right-hand array of boxes. [For instance, if that date were March 29, you would enter either 3 or 03 in the left-hand boxes and 29 in the right-hand boxes.]

Difficulty: standard-h. This problem is a good classroom problem, but it would not be appropriate for an MCA. Reason: If students have seen the problem before, they might remember the answer 07-02. It is great if students have learned a methodology in the classroom that happens to be useful on an MCA test aligned with the standards. That is quite different from happening to have memorized the interesting answer to an interesting question that might have been treated in some, but not all, classrooms. There is another issue with this problem as an MCA problem; some recent immigrants come from a culture in which the date is written in front of the month. However, this issue is not critical; the grading could give full credit for either 07-02 or 02-07 (and also for modifications such as 7-2 or 02-7).

A pound of cotton has been spun into a thread 8 miles in length. Allowing for 235 pounds of waste, how many pounds will it take to spin a thread to reach around the earth, supposing that distance to be 25,000 miles?

Difficulty: standard-h. [This problem comes from a grades-5-8 book first published in 1858 and reprinted in 1863 and 1877 (which is the date of my copy). This book was authored by Daniel W. Fish and is entitled 'Robinson's Progressive Practical Arithmetic.'] Note: It is obvious, from reading all the words after the first word in benchmark 5-II.B.5, that the first word of that benchmark should be 'Divide' not 'Multiple'.
FOR GRADE 6, WITH COMMENTS

6-II.B.8  What percentage of the following numbers are greater than $-3\frac{2}{5}$:
6-II.A.1

$-7, \ 7, \ -\frac{7}{2}, \ \frac{7}{2}, \ -3.72, \ 3.72, \ -3, \ 3, \ -\frac{2}{7}, \ \frac{2}{7}$?

Difficulty: standard-h.

5-IV.A.4  Find the mean of the eight numbers $-10, -7, -5, -2, -1, 0, 3,$ and 4. Write your answer as a fraction, possibly improper, in lowest terms. Then enter the numerator in the left-hand boxes and the denominator in the right-hand boxes; in each of these arrays of boxes there is sufficient room to insert a negative sign if needed.

5-II.A.3

Difficulty: standard-h. Although the mean appears in Grade 5, arithmetic with negative numbers only appears in the Grade-6 standards. (Compare standard 5-II.B with standard 6-II.B.) When treating ‘mean’ in Grade 5, a teacher will be working in a milieu where it is very natural to treat positive and negative fractions with big denominators, so that students feel comfortable with them in Grade 6. I note that even though this problem is a machine-graded problem, one could arrange—and I think one should—for partial credit in case the answer is not in lowest terms. Also, it is important that the grading mechanism be able to handle the correct answer whether the needed negative sign is attached to the numerator or to the denominator.
A man purchased a house for $118,750, and expended $17,000 in repairs; he then sold it for railroad stock worth $43,350, and 235 acres of western land valued at $400 an acre. What dollar amount did he gain by the trade?

Difficulty: standard-h. I would regard this problem as inappropriate for an MCA. The colorful adjective ‘western’ for the noun ‘land’ can spark the imagination of some, while being nothing but a puzzle for others. But it is fine for in-class discussion. [This problem comes from the ante-bellum book by Fish mentioned earlier, except that the dollar amounts here have been obtained by multiplying those in the book by 50.]
If $5 \frac{1}{2}$ yards of cloth are required for a coat, $3 \frac{1}{2}$ yards for a matching pair of pants, and $\frac{7}{9}$ yards for a vest, how many yards will be required for all three items of clothing? Write the answer as a mixed number with the fractional part in lowest terms. Enter the numerator of the fractional part in the left-hand boxes and its denominator in the right-hand boxes.

Difficulty: standard-h. Actually, this problem is inappropriate for any MCA for two reasons: (i) The non-mathematical word ‘vest’, which plays no essential role in the problem, is a word which might not be familiar to certain students. (ii) One would think that an appropriate unit for cloth is one that measures area. ‘Yards’ measures length, not area. The problem becomes clear only when one knows that length in one direction is what is important since the cloth comes off of a reel of standard width. This second reason for the problem not being appropriate for an MCA happens to be a good reason for using the problem in a class discussion. Also, there is the issue of whether $1/9$ and $7/9$ would be called common fractions as specified in benchmark 6-II.B.7. My judgment is that they should be. Clearly benchmark 6-II.B.7 does encompass the concept of common denominator. The restrictions in the benchmarks are there to limit the computation needed to find a common denominator. In this problem, that computation is indeed limited. This is a problem for which partial credit could be given for a correct answer not in lowest terms. [This problem comes from the ante-bellum book by Fish mentioned earlier.] A personal comment: Mastery of the benchmarks 6-II.B.1 and 6-II.B.2 give students the power to handle addition and subtraction of fractions having a wide variety of denominators, but it is not until Grade 7 that the standards indicate mastery in this area. This scheduling of topics seems reasonable. For instance, it might be near the end of the school year in Grade 6, after the Grade-6 MCA is given, when a student is asked to become adept at applying her or his skill with prime factorization to the addition and subtraction of fractions, and, moreover, there are Grade-7 standards which would certainly reinforce techniques for adding and subtracting fractions.
FOR GRADE 6, WITH COMMENTS

5-IV.A.4 During one June, the precipitation amounts in Phoenix, Arizona were 0.4 inches during the first ten days of June, 1.3 inches from June 11 through June 20, and 0.7 inches during the last ten days of the month. Calculate the mean daily rainfall correct to 3 places to the right of the decimal point.

6-I.3
6-I.5
6-II.B.4

Difficulty: standard-h. This problem requires use of the definition of mean in a setting where individual data are not given. The student has to recognize that nevertheless there is sufficient information to do the problem.

6-II.B.2
6-II.B.4

Find the greatest common divisor of 294 and 441.

Difficulty: substantial. If a person decides to look for prime factors, three of them have to be found: 3 and two 7's. There are shortcuts, but knowledge of these should not be expected of all students. A calculator version of this problem might be to find the greatest common divisor of 67, 507 and 79, 781. Even with the calculator this problem is probably more difficult than the one above without a calculator.

6-II.B.7
6-II.A.1

Calculate \( \frac{5}{6} \times \frac{2}{5} \).

Write the answer as a mixed number with the fractional part in lowest terms. Then place the numerator of the fractional part of the answer in the left-hand boxes and the denominator of the fractional part in the right-hand boxes.

Difficulty: substantial. I would think partial credit would be warranted if all but the reduction to lowest terms is correct. Also, a small amount of partial credit might be given for the correct denominator; the correct denominator combined with an incorrect numerator indicates that the problem was done correctly except for the conversion of the final improper fraction into a mixed number.
6-V.B.1 Suppose that three rays emanate from a common point and that the angle formed by the middle ray and one of the outside rays is a right angle, and that the measure of the angle formed by the middle ray and the other outside ray is 1/2 that of a right angle. What is the measure in degrees of the angle formed by the two outside rays?

Difficulty: substantial. Were this problem posed with a picture rather than an involved verbal description, its level of difficulty would be less. An issue for Minnesota: How should ‘measures of angles’ be handled. The source of the issue is this. For some, an angle consists of two rays emanating from a common point; for others, it is a number of degrees; for still others, it is used in both ways, even in the same sentence. The issue doesn’t arise for line segments; there we have another term ‘length’ for its measure. It is important that the State handle such issues in a way that doesn’t create artificial hurdles for certain students. A carefully designed information sheet could be sent to the schools on issues such as this. Teachers could use such information in constructive ways as they teach their classes.

6-I.3 Points A and B are 17.2 miles apart. John bicycles from A to B at an average speed of 6.7 miles per hour and returns from B to A at an average speed of 4.3 miles per hour. What is John’s average speed in miles per hour for the round trip: A to B and back to A? Round your answer to the nearest 0.1 miles per hour.

Difficulty: challenging. This problem requires careful thought and, moreover, it requires by-hand division at the limit of what is specified in the benchmarks. The reason for listing the benchmark 6-I.1 is that a natural error which one can make in this problem gives an answer that is twice the correct answer, but a student should be aware that an answer larger than 6.7 cannot possibly be correct.
6-V.B.1 In the following picture, the line through the points $A$ and $D$ also passes through the point $C$. In the left-hand triangle the vertices at $B$ and at $A$ each have measure $25^\circ$. In the right-hand triangle the vertices at $B$ and at $C$ have the same measure as each other. Calculate the measure in degrees of the vertex at $D$. The picture has not been drawn accurately.

Difficulty: challenging.

6-V.B.2

6-I.4

A person does a long division problem dividing 57 into some larger integer. This person makes no mistakes in addition, subtraction, and multiplication of integers and ends up with a remainder of 75. This remainder would usually be considered to be unsatisfactory because usually the goal is to have a nonnegative remainder that is less than the divisor. With this goal in mind, what should this person have obtained as a remainder?

Difficulty: challenging. Although the student only needs fill in the answer, I think benchmark 6-I.4 is relevant for it is hard to see how a person can do this problem correctly without having a well-formulated understanding of the situation in mind.
6-V.A.1 Sketch a two-dimensional figure that can be folded into a triangular prism—the top and the bottom being triangles and the three sides being rectangles (possibly squares). Show the fold lines, but you do not have to include tabs for gluing.

Difficulty: standard-h. A figure such as the one requested is sometimes called a ‘net’. It is important that there be communications from the state, far in advance of MCA’s, concerning terminology that is sometimes used but which would not necessarily be in the background of students who have nevertheless learned the concepts in the standards document. The problem would be more difficult were it to request labeling of the vertices consistent with a labeling of the vertices of the prism.

6-IV.B.2 A deck consisting of four cards labeled A, B, C, and D is shuffled and then the top two cards are drawn in order. Make a list describing the possible outcomes of this experiment. Then calculate the probability that A is the first card or D is the second card.

5-IV.B.1 6-I.6

Difficulty: substantial. The first part of this problem is appropriate at Grade 5, but the second part would not be appropriate at Grade 5 and even at Grade 6 is a bit subtle.
6-II.B.1 Calculate the prime factorization of 6,438,000. Show your work and make your reasoning clear.

Difficulty: challenging.
The following problems are designed with the **calculator portion of the Grade-6 MCA** in mind.

### 6-II.B.9
Find

\[(27.314 + 15.337) \times 2.1223.\]

The best 5-digit approximation of the answer equals

(a) 59.863
(b) 59.864
(c) 90.518
(d) 90.519

Difficulty: standard-e. One could consider giving partial credit for response (d).

### 6-II.B.9
With accuracy in two places to the right of the decimal point,

\[7.77 \times [3329.23 - (45872.1 - 955.25)]\]

equals

(a) \(-337,980.39\)
(b) \(-323,135.81\)
(c) \(-20,959.23\)
(d) \(-19,048.71\)

Difficulty: standard-h.

### 6-II.B.9
Mr. Samuel Jones buys 5 shirts at $17.37 each, 12 pairs of socks at $4.65 per pair, and two winter caps at $9.52 each. What is the total amount of money that Mr. Jones spends? Enter the whole number of dollars in the left-hand boxes and the number of cents in the right-hand boxes.

Difficulty: standard-e.
FOR GRADE 6, WITH COMMENTS

6-V.B.3  The circumference of a certain circle equals 7 feet, 5 inches. Calculate its radius to the nearest inch. You may use the approximation 3.14 for $\pi$ or you may use the key for $\pi$ itself on your calculator.

Difficulty: standard-h. One might want to consider giving partial credit for finding the diameter rather than the radius. I would not, however, give partial credit for acting as if the area rather than the circumference has been given.

6-V.C.1  One mile equals 5280 feet. To the nearest one-hundredth of a mile, how many miles are 205,321 feet?

Difficulty: standard-h. Some partial credit might be warranted for 38.88 even though the correct answer is 38.89.

6-II.B  Calculate the following sum to the nearest one-hundredth:

$$4.592 + - 3.4449 + 4\frac{7}{17} + - 2\frac{5}{7}$$

Difficulty: challenging. The presence of fractions in some numbers and decimals in others creates a complication. At the Grade-6 level, negative numbers also provide an additional hurdle. There is a notational issue connected with negative numbers, especially at Grades 6 and 7: Should the test distinguish between the minus symbol and the negative symbol by the height at which ‘$-$’ is printed?
A 4-faced die with the faces labeled as 1, 2, 3, and 4 is rolled 5128 times. [It is not known whether the die is well-balanced.] The results are:

1 occurred 1013 times
2 occurred 1380 times
3 occurred 1502 times
4 occurred the other times

Make an accurate bar graph showing the numbers of times that each of the four numbers occurred.

Difficulty: standard-e.

The views and opinions expressed in this link are strictly those of Bert Fristedt. The contents have been neither reviewed nor approved by the University of Minnesota.