The purpose of this link from my web-site is to identify a selection of problems aligned with the Minnesota mathematics standards and benchmarks for Grades 9-12 as adopted in Spring 2003. My focus consists of the standards and benchmarks themselves; the problems here serve to illuminate them. The benchmarks and standards that are particularly relevant for a particular problem are identified in the left-hand margins; in Part I for instance, 9-11-IV.A.2 indicates the Grades-9,10,11 benchmark IV.A.2 and 9-11-IV.A refers to the corresponding standard. In another sense, the focus in Part I is the suitability of problems for the Minnesota Comprehensive Assessments (know as MCA’s), but in saying this I want to emphasize that the opinions are mine alone, formed without consultation with Minnesota Department of Education. Part II of the document only includes problems for which at least one of the relevant benchmarks or standards is an 11-12 benchmark or standard. There are three 11-12 standards—Statistics, Algebra, and Trigonometry & Geometry—and these will be abbreviated by St, Al, and TG, respectively. Thus, 11-12:Al.13 denotes benchmark 13 under Algebra for Grades 11-12, and 11-12:Al denotes the corresponding standard. For ease of comparison with 9,10,11 (which can also be written as 9-10-11 when there is no danger of too many hyphens), the format of the problems in Part II is the same as in Part I even though there will be no MCA’s related to Part II. Relevant to the above concerns is a companion link which includes the problems from this link, along with a variety of comments.

I was one of approximately 40 members of the mathematics subcommittee of the Academic Standards Committee, formed by the Minnesota Commissioner of Education in February 2003. I strongly support the mathematics standards and benchmarks resulting from the work of that committee and which, on the basis of a law passed by the Legislature and signed by the Governor, became official in Spring 2003. Although there is no guarantee that this web-site item reflects the thinking within the Department of Education, I have tried very hard to reflect the standards and benchmarks accurately, taking care not to bend them in the direction of my individual views. [Even though I strongly support the standards and benchmarks document, there are places where I would have preferred the document to be a bit different, and I suspect that the same is true (but not for the same places) of every member of the mathematics subcommittee.]

Anticipating that I might want to modify this document from time to time, I have refrained from labeling the problems with numerals and am planning to change the date at the top any time I make additions or changes.
Since the standards are cumulative, all the K-11 benchmarks are relevant for the Grade-11 MCA. It seems to me that it is desirable for Grades 9, 10, 11 teachers to examine all the K-12 benchmarks giving special attention to those for grades 7-12, and in general for teachers to read the standards for a couple grades on either side of the grade they are teaching. Since the content related to some of the 11-12 benchmarks can at least be introduced in conjunction with a focus on 9-10-11 benchmarks, it is important that the 11-12 standards and benchmarks be read by all high school mathematics teachers.

The variety of different problems that are consistent with the standards and benchmarks is very large—that is the power of mathematics; a manageable number of basic principles and techniques enables one to handle a myriad of different situations. So, of course, the problem list that follows cannot be viewed as comprehensive.

For problems in which students are to place the correct digits in boxes, a decimal point or comma is included between appropriate pairs of boxes when relevant. If the answer requires fewer digits than boxes, it is the left-hand box or boxes which should be left blank. [If the Grade-11 MCA were, in fact, to include such problems it would be important that students become familiar with the instructions some days in advance of the test.]

There is not a sharp demarcation separating problems appropriate for various grade levels. For instance, some of the problems described below for Grades 9-10-11 are also in the link for Grade 8. Typically, a problem that is appropriate for both the Grade-11 MCA and the Grade-8 MCA would be regarded as a more difficult problem for an eighth grader than it would be regarded for a high school student.

A graphing calculator is not mentioned in the 9-12 standards and benchmarks. My own view is that the 9-10-11 standards can be tested without a graphing calculator being involved, but that it would also be consistent with these standards for the Grade-11 MCA to have a few problems for which a graphing calculator would be essential. In this connection, there is an expense issue that has to be considered. I have chosen to include some graphing-calculator problems in the calculator-permitted portions of this document. The first portion of each part of this document begins with calculator-prohibited problems. Later in each part, an introductory sentence identifies the place where the ‘calculator permitted’ portion begins.

I want to again emphasize: Although the standards and the benchmarks accompanying them constitute an official document of the state of Minnesota, all the judgments about alignment of problems with the benchmarks and standards are mine; neither do they have any official standing nor have they been obtained in consultation with the Minnesota Department of Education. Also, they have not been reviewed by the University of Minnesota where I am a faculty member and, of course, they do not represent any official view of that institution.
8-V.B.2  On a drawing of the floor plan of a particular house, one foot in the house is represented by 1/8 of an inch. How long is the actual house if the length as measured on the drawing is 4 3/4 inches?
   (a) 17 feet 6 inches
   (b) 19 feet
   (c) 35 feet
   (d) 38 feet

7-II.B.4  What does 5% of 2% equal?
   (a) 0.1%
   (b) 0.4%
   (c) 10%
   (d) 40%

7-IV.B1  To the nearest whole percent, what is the probability, in percentages, of rolling a three with a perfectly balanced die?
6-II.A.2
7-II.A.1  (a) 6.7
   (b) 16
   (c) 16.7
   (d) 17
A ball made out of a certain material weighs 32 pounds and has a diameter of 6 inches. Another ball of the same material weighs 4 pounds. What is its diameter in inches?

(a) $\frac{3}{4}$
(b) $3/\sqrt{2}$
(c) $\frac{3}{2}$
(d) 3

Simplify the expression $x^2(x^3 - 5)$.

(a) $x^5 - 5x^2$
(b) $x^5 - 5$
(c) $x^6 - 5x^2$
(d) $x^6 - 5$
What is the area of the pentagon shown below?

(a) 78
(b) 85
(c) 86
(d) 100
FOR GRADES 9-12, WITHOUT EXTRA COMMENTARY

PART I (without-calculator portion related to 9-10-11 standards)

9-11-V.B.1 Which of the following does not represent a criterion for two triangles to be congruent?
   (a) angle-side-angle
   (b) side-angle-side
   (c) side-side-angle
   (d) side-side-side

9-11-V.B.3 Let $h$ and $r$ denote the height and radius, respectively, of a right circular cylinder closed on both the bottom and the top. Which of the following is a formula for its surface area?
   (a) $\pi r^2 h$
   (b) $\pi (r^2 + rh)$
   (c) $\pi (r^2 + 2rh)$
   (d) $2\pi (r^2 + rh)$

9-11-III.B.9 Find $b_7$ for the sequence defined recursively by

\[
\begin{align*}
   b_1 & = 5 \\
   b_n & = b_{n-1} + 3 \quad \text{for } n > 1 .
\end{align*}
\]

   (a) 23
   (b) 26
   (c) 33
   (d) 38
If you start to solve the following system of equations by either eliminating $y$ or substituting for $y$, what is the equation you obtain for $x$?

\[
\begin{align*}
5x - 2y &= 7 \\
2x + y &= -4
\end{align*}
\]

(a) $x = 15$
(b) $3x = 11$
(c) $7x = 3$
(d) $9x = -1$

Only one of the following four equations represents a function of $x$. Which one?

(a) $4x^2 + y^2 = 4$
(b) $4x^2 - y^2 = 4$
(c) $y^2 = x + 7$
(d) $x^2 = y + 7$

Let $k$ and $l$ be two parallel lines in plane geometry. Let $m$ be a line parallel to $k$ and $n$ a line perpendicular to $k$. Which of the following is then true?

(a) $m$ is parallel to $l$ and $n$ is perpendicular to $l$.
(b) $m$ is perpendicular to $l$ and $n$ is parallel to $l$.
(c) $m$ and $n$ are each parallel to $l$.
(d) $m$ and $n$ are each perpendicular to $l$. 
Which of the following is the graph of the equation \( y = 3|2 - x| - 1 \)?

(a) (b)

(c) (d)
9-11-V.B.1 A reminder of a definition: A parallelogram (in plane geometry) is a quadrilateral whose opposite sides are congruent and parallel to each other. Here is a theorem.

9-11-I.4 In a parallelogram $ABCD$ with sides $AB$, $BC$, $CD$, and $DA$, the angles at $B$ and $D$ are congruent to each other. Here is a proof of this theorem, with, however, an adjective missing from the proof. The line segment $AC$ is common to the two triangles $\triangle ABC$ and $\triangle CDA$. Thus, we can use the criterion for congruence to conclude that $\triangle ABC \cong \triangle CDA$. Therefore the angle at $B$ is congruent to the angle at $D$. Which of the following adjectives should be inserted in the blank?

(a) angle-side-angle
(b) side-angle-side
(c) side-side-angle
(d) side-side-side

9-11-III.B.5 A certain quadratic function of $x$ represents the net profit that can be earned when $x$ is the number of dolls that are made in anticipation of the Christmas shopping season. When $x = 30,000$, the value of the function in thousands of dollars is 500. When $x = 40,000$, the value of the function is 650. All that is known about the value of the function at $x = 50,000$ is that it is less than 650. The number of dolls that will produce the maximum profit is

(a) definitely between 35,000 and 45,000.
(b) definitely between 35,000 and 50,000, and it is possible that it is 45,000 or more.
(c) definitely between 30,000 and 45,000, and it is possible that it is 35,000 or less.
(d) none of the above.
FOR GRADES 9-12, WITHOUT EXTRA COMMENTARY

PART I (without-calculator portion related to 9-10-11 standards)

9.11-IV.A.4 Let \( \mu \) and \( m \) denote the mean and median, respectively of some odd number of data. Now suppose two new data are adjoined to set of data and that one new datum is much larger than both \( \mu \) and \( m \) and the other datum is slightly smaller that \( \mu \) and also smaller than \( m \). The effect of incorporating these two new data into the calculations is to:

(a) lower the mean and keep the median the same
(b) raise the mean and keep the median the same
(c) lower both the mean and median
(d) raise both the mean and median

8-II.A.1 Which of the following statements is true?
8-II.B.4

(a) \( \sqrt{2} < \frac{41}{29} < \frac{99}{70} \)
(b) \( \sqrt{2} < \frac{99}{70} < \frac{41}{29} \)
(c) \( \frac{41}{29} < \sqrt{2} < \frac{99}{70} \)
(d) \( \frac{41}{29} < \frac{99}{70} < \sqrt{2} \)

9.11-IV.B.2 When two fair dice are rolled, what is the probability that at least one of the two dice will show a five?
8-I.6
9.11-I.3

(a) 1/36
(b) 1/12
(c) 11/36
(d) 1/3
9.11-V.B.6 To where does the point \((\sqrt{3}, 1)\) go under a counterclockwise rotation through an angle of 120°? [For this problem, assume that the axes are in the usual positions so that a counterclockwise rotation through an angle of 90° takes points having positive first coordinate to points with positive second coordinate.]

\[
\begin{align*}
(a) & \ (-\sqrt{3}, 1) \\
(b) & \ (\sqrt{3}, -1) \\
(c) & \ (1, -\sqrt{3}) \\
(d) & \ (-1, \sqrt{3})
\end{align*}
\]

9.11-V.B.3

Suppose \(x, y,\) and \(z\) are related by the formula \(z = xy^2\). Find \(x\) when \(y = 3\) and \(z = 117\).

9.11-III.B.7

The formula for the surface area of a rectangular parallelepiped in which two faces are squares is \(2(x^2 + 2xy)\), where \(x\) is the edge length of the square faces and \(y\) is the length of the other edges of the parallelepiped. Find \(y\) when the surface area equals 410 and \(x = 5\). \(Hint:\) The answer is a whole number.

6-V.B.1

Suppose that three rays emanate from a common point and that the angle formed by the middle ray and one of the outside rays is a right angle, and that the measure of the angle formed by the middle ray and the other outside ray is 1/2 that of a right angle. What is the measure in degrees of the angle formed by the two outside rays?
FOR GRADES 9-12, WITHOUT EXTRA COMMENTARY

PART I (without-calculator portion related to 9-10-11 standards)

8-IV.B.2  The odds against a certain event occurring are 7 to 4. Find the best 3-place decimal approximation of the probability that this event occurs.

7-II.B.4

7-II.A.1

9-11-III.B.3  Find an equation of the line passing through the points (3, 2) and (−1, 3), and write the equation in the form

\[
\frac{x}{a} + \frac{y}{b} = 1
\]

for appropriate constants \( a \) and \( b \). (Here \( x \) denotes the first coordinate and \( y \) denotes the second coordinate.) Write \( b \) as a fraction in lowest terms and then enter its numerator in the left set of boxes and its denominator in the right set of boxes. [Do not enter \( a \) on the answer sheet.]

9-11-III.B.4

8-II.B.4  Solve the quadratic equation

\[
2x^2 - 8x + 1 = 0
\]

by using the quadratic formula. Write your answer in the form

\[
\frac{4 \pm \sqrt{w}}{2}
\]

and enter the value of \( w \) in the boxes.
Joachim has a certain number of quarters together with 3 nickles. Allison has the same number of dimes as Joachim has quarters, and in addition she has 54 nickles. These coins account for all the money that these two people have, and, moreover, they have the same amount of money. How many dimes does Allison have?

Rewrite the expression

\[3(5x - 2) + 2(x - 1)\]

in a simplified form not requiring parentheses.

Solve the inequality

\[4 < 2x - 3 < 7\]

and sketch a picture of the solution set on the real line.

Find an equation of the line that passes through the point \((3, -8)\) and is perpendicular to the line \(y = 5x - 7\). Write your answer in slope-intercept form.
9.11-III.B.1  Simplify the following expression to a form that does not require parentheses:

$$3x^2[2x(x - x^2) + x^3] - 8x^4.$$ 

9.11-IV.B.6  Describe how to use a regular well-balanced six-faced die to simulate an experiment in which each of the numbers 1 and 2 is to occur with probability $\frac{1}{3}$ and each of the numbers 3 and 4 is to occur with probability $\frac{1}{6}$.

9.11-IV.A.2  A teacher gives two one hundred-point tests to her class. On both of them the mean score is 65. On one of them the first quartile is at the score 25 and the third quartile is at the score 90. On the other test the first quartile is at the score 50 and the third quartile is at the score 76. What might this teacher conclude about the difficulties of the questions she has placed on the two tests, from the point of view of comparing the two tests?
A well-balanced spinner has five colored sectors, the colors being red, orange, yellow, green, and purple. The angle measurements are such that the probabilities of the various colors on a single spin are

\[
\frac{1}{3} \text{ for red, } \frac{1}{4} \text{ for orange, } \frac{1}{6} \text{ for yellow, } \frac{1}{6} \text{ for green, } \frac{1}{12} \text{ for purple.}
\]

The spinner has been spun 24 times (assume independence among the various spinnings). The number of occurrences of the various colors has been recorded as follows

- 7 on red
- 8 on orange
- 4 on yellow
- 3 on green
- 2 on purple

Use a protractor to draw a circle graph (sometimes called a pie chart) representing the 24 outcomes in terms of percentages totaling 100% for the five colors. Also show a table indicating the angular degrees of each sector for this circle graph. Of course, the spinner itself is a circle graph for the actual probabilities themselves. Explain why one might prefer bar graphs rather than circle graphs for the purpose of comparing the actual probabilities with the percentages of occurrences. [Were this problem to be given on a state test, a reasonably accurate picture of the spinner should be shown and the requested table should be given missing only the entries of the degrees.]
The names of the shrubs and bugs in this problem are fictional. In a certain large collection of goaple shrubs it has been found that some of these shrubs live to the age of approximately 12 years whereas others die before the third year, and this is true regardless of the years when they are planted and the climatic variations from year to year. It has been found that there are many sherky bugs in those dead shrubs that have died early, but not in those shrubs that lived for over 10 years before dying. One is tempted to conclude that sherky bugs are a significant cause of death to goaple shrubs. Create a scenario which fits the above data, but which is inconsistent with the conclusion mentioned in the preceding sentence.
Here is a method for constructing the angle bisector of a given angle. With the compass draw a circle centered at the vertex of the given angle, and keep the radius of the compass fixed for the next step. Then draw two further circles having the same radius with the centers of these two circles being the points of intersection of the first circle with the two sides of the given angle. These latter two circles intersect at the vertex of the given angle and at one other point. With a straightedge draw the ray passing through this other point and emanating from the vertex of the given angle. Prove that this ray is the angle bisector.
When using the Borda Count Method in an election with $n$ candidates, the first choice of a voter gets $n$ points, the second choice gets $(n - 1)$ points, and so forth.

Construct an example involving four candidates in which one candidate gets a majority (not just a plurality) of first-place votes and some other candidate wins on the basis of the Borda Count Method.
PART I (without-calculator portion related to 9-10-11 standards)

9.11-V.B.2 Prove that the two tangent line segments from a point outside a circle to that circle are congruent. You may use the fact that a tangent line to a circle is perpendicular to the radius of the circle meeting it at the point of tangency.

9.11-V.B.3 A triangle is inscribed in a circle of radius 5 in such a way that one side of the triangle is a diameter of the circle and the altitude to that side from the opposite vertex meets that side at a point a distance 2 from one endpoint of the diameter as shown in the figure below. Find the area of the triangle.
The following problems are designed with the calculator portion of the Grade-11 MCA in mind.

9-11-IV.B.4 A certain unbalanced die comes up $k$ with probability $k/21$ for $k = 1, 2, 3, 4, 5, 6$. Calculate the expected value for one role of the die.

(a) $3\frac{3}{4}$
(b) $4\frac{1}{3}$
(c) $4\frac{3}{4}$
(d) $5\frac{2}{8}$

9-11-V.B.4 Find the length of the hypotenuse of a right triangle accurate to four significant digits if one leg of that right triangle has length 7 and the angle formed by that leg and the hypotenuse has measure $67^\circ$.

(a) 2.735
(b) 7.605
(c) 16.49
(d) 17.92

9-11-IV.B.1 How many three-digit numbers are there which have no digit equal to 0?

9-11-I.3

(a) 504
(b) 729
(c) 900
(d) 997

9-11-III.B.8 The following quadratic equation has a solution satisfying one of the four given inequalities. Which inequality?

$5.345x^2 - 7.824x + 2.125 = 0$

(a) $1.09 < x < 1.10$
(b) $1.10 < x < 1.11$
(c) $1.11 < x < 1.12$
(d) $1.12 < x < 1.13$
FOR GRADES 9-12, WITHOUT EXTRA COMMENTARY

PART I (calculator-permitted portion related to 9-10-11 standards)

9.11-III.A.2 For each of the four functions below, three of its values are given. On the basis of this information alone, one of the functions can be determined to definitely not be an exponential function. Which one?

(a) \( f(2) = 4, \quad f(4) = 8, \quad f(8) = 16 \)
(b) \( g(2) = 4, \quad g(4) = 1, \quad g(6) = \frac{1}{4} \)
(c) \( h(2) = \sqrt{2}, \quad h(4) = 2, \quad h(8) = 8 \)
(d) \( k(2) = \sqrt{2}, \quad k(4) = 1, \quad k(6) = \frac{1}{2} \)

9.11-III.A.4 If $100 is invested at 4% annual interest rate, compounded annually, what is the value of the investment (including the original $100) after 10 years? Write your answer to the nearest cent.

9.11-V.B.3 Find the length, to the nearest whole number, of the long diagonals of a rectangular parallelepiped whose side lengths are 42, 118, and 135. [Note: By ‘long diagonal’ is meant a line segment connecting two vertices of the parallelepiped, but not lying in any one face of the parallelepiped.]

9.11-II.B.1 The graphs of the functions \( y = \sqrt[3]{2x^3 - 1} \) and \( y = 5 - 3x \) meet at exactly one point. Find the \( x \)-coordinate of that point accurate to two decimal places.

9.11-II.B.2

9.11-III.B.5
In a certain quadrilateral $ABCD$ (in plane geometry):

- the angles at $B$ and $C$ are right angles;
- the length of the side from $C$ to $D$ equals 5;
- the length of the side from $B$ to $A$ equals 3;
- the length of the side from $B$ to $C$ equals 3.

Find an approximation to the nearest degree of the angular measure at $A$.

Find a formula for the function of $x$ that gives the distance of the point $(x, x^2 - 3)$ from the point $(0, 0)$ in a plane.
For a situation where it is known that $y$ is a linear function of $x$, four data are obtained with experimental error, but which, if there had not been experimental error would lie on the line representing that linear function. The four data for $(x, y)$ are:

$$(3.12, 1.44), \ (4.03, 2.51), \ (5.42, 4.01), \ (5.90, 4.60).$$

Use these experimental points to calculate six two-decimal-place estimates of the slope of the linear function, and then find the median of these six estimates.
The following problems involve the 11-12 standards, at least in part. Thus none of these problems would, in my opinion, be appropriate for a Grade-11 MCA.

11-12:Al.7 \[ \log_2 8 = \]
   (a) 1/3
   (b) 3
   (c) 64
   (d) 256

11-12:TG.1 Which of the following is a correct statement whenever the denominator on the left-hand side is defined and different from zero?
   (a) \(\frac{\cos \theta}{\sin \theta} = \tan \theta\)
   (b) \(\frac{1}{\sin \theta} = \sec \theta\)
   (c) \(\frac{1}{\sin \theta} = \cos \theta\)
   (d) \(\frac{1}{\tan \theta} = \cot \theta\)

11-12:TG.2 For the angle with vertex (0,0) and sides passing through (1,0) and (5,8), calculate its secant.
   (a) \(\sqrt{39}/8\)
   (b) \(\sqrt{89}/8\)
   (c) \(\sqrt{39}/5\)
   (d) \(\sqrt{89}/5\)

11-12:Al.15 Suppose that \(f\) is a one-to-one function for which \(f(0) = -3\), \(f(1) = -2\), \(f(2) = 0\), and \(f(3) = 2\). Denote the inverse function of \(f\) by \(f^{-1}\). Find \(f^{-1}(2)\).
   (a) -3
   (b) 0
   (c) 1/3
   (d) 3
Which of the following is a correct trigonometric identity?

(a) \( \sin 2\theta = 2 \sin \theta \cos \theta \)
(b) \( \sin 2\theta = \frac{1}{2} \sin \theta \cos \theta \)
(c) \( \sin 2\theta = 2 \sin^2 \theta - 1 \)
(d) \( \sin 2\theta = 1 - 2\sin^2 \theta \)

The product obtained by multiplying the square root of a positive number \( b \) by the square of the cube root of \( b \) equals

(a) \( b^{1/3} \)
(b) \( b^{2/3} \)
(c) \( b^{5/6} \)
(d) \( b^{7/6} \)

Using the table provided (the table is not on this web site) calculate the probability to three decimal places that a random datum satisfying the normal distribution will lie within 1.75 standard deviations of the mean.

Find the coefficient of \( x^3y^9 \) in the expansion of \( (x + y)^{12} \). Make sure you simplify your answer.
Find an integer $k$, different from 0, such that the following quadratic equation has exactly one solution:

$$21x^2 - 3kx + 33k = 0.$$ 

The expression $6x^2 + 11x - 10$ has two factors of the form $ax + b$ for integers $a$ and $b$. For either of these two factors, enter the value of $a$ in the left-hand boxes and the value of $b$ in the right-hand boxes. [There are two distinct correct responses to this problem; either one is ok.]

Find all solutions of the following system of equations:

$$\begin{cases} 
    x + 2y - 8z = 3 \\
    x + y - 3z = 1 \\
    3x + 2y - 4z = 1 
\end{cases}$$

If there is only one solution, enter the value of $x$ for that solution in the boxes. If there is more than one solution, enter the value of $x$ for the solution in which $z = 15$.

For a certain constant $c$, the line $x + 3y = c$ meets the circle $x^2 + y^2 = 12$ in exactly one point. Find $c^2$. 

26
11-12:Al.10 Factor

\[ 9x^2 - 13y^2. \]

11-12:Al.14 Carefully state the theorem that relates factors of a polynomial \( p(x) \) to solutions of an equation of the form

\[ p(x) = 0. \]

9-11-I.6 Sketch the graph of

\[ 9x^2 - 4y^2 + 8y = 0 \]

by first rewriting the given equation in an appropriate equivalent form. Also, find both vertices exactly, as well as equations for the asymptotes in case there are any.
FOR GRADES 9-12, WITHOUT EXTRA COMMENTARY

PART II (without-calculator portion related to 11-12 standards)

11-12:St.2 Explain how the bell-shaped curve is involved in the central limit theorem. It is not required that you give a precise statement of the central limit theorem.

11-12:TG.8 Find all solutions between 0 and $2\pi$ radians of the equation

$$\sin \theta = \cos^2 \theta.$$ 

You may leave an inverse trigonometric function in your answer.
Fill in the two blanks in the following sentence and prove that you have done so correctly. For \( x \neq 0 \),

\[
\arctan x + \arctan(1/x) = \underline{\text{_______}} \text{ or } \underline{\text{_______}}.
\]

[In this problem \( \arctan \) denotes what is sometimes called the 'principle value of the inverse tangent'.]

Find a fourth-degree polynomial

\[
b_0x^4 + b_1x^3 + b_2x^2 + b_3x + b_4
\]

which has the value \( \cos 4\theta \) when \( x = \cos \theta \). \textit{Hint:} You may use the identity

\[
\cos 2\theta = 2\cos^2 \theta - 1.
\]
In the following problems, it would be expected that a calculator, and in one case a graphing calculator, would be available.

**11-12:TG.3** Convert 73 degrees to radians, correct to three significant digits.

\[
\text{rad} \approx \frac{73 \times \pi}{180}
\]

**11-12:TG.4** Two vertex angles of a triangle have measures of 41 degrees and 68 degrees, and the length of the side between them is 687. Find, to the nearest integer, the length of the side opposite the 41-degree angle.

\[
x = \frac{687 \times 68}{\sin 68^\circ} 
\]

**9-11-I.4**

**9-11-II.A.2** A certain amount of money is deposited in an account for 16 years at a constant annual interest rate compounded continuously. At the end of 16 years, the amount in the account is three times the initial deposit. What is the annual interest rate in percentages to three significant figures?

\[
\text{rate} = \frac{\ln 3}{16} \times 100 \%
\]