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Complex analysis examples 01

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[This document is http://www.math.umn.edu/˜garrett/m/complex/examples_2014-15/cx_ex_01.pdf]

If you want feedback from me on your treatment of these examples, please get your work to me by Friday, Sept 12, preferably as a PDF emailed to me.

[01.1] Express the two values for $\sqrt{i}$ in terms of radicals.

[01.2] Determine all values of i^i.

[01.3] Derive the usual formula for $\sin(z+w)$ by using e^z.

[01.4] Express $\cos 5x$ as a polynomial in $\cos x$ and $\sin x$.

[01.5] By *mere algebra*, write a power series expansion near $z=0$ for

$$f(z) \;=\; \frac{1}{(z-1)(z-2)}$$

[01.6] Determine the radius of convergence of $\sum_{n\ge 1} \frac{3^n}{n(n+1)(n+2)}\, z^n$.

[01.7] Determine the radius of convergence of $\sum_{n\ge 1} \frac{n!}{n^n}\, z^n$.

[01.8] For two complex numbers a, b, with b not a non-positive integer, show that the radius of convergence of

$$\sum_{n\ge 0} \frac{a(a+1)(a+2)\ldots(a+n-1)(a+n)}{b(b+1)(b+2)\ldots(b+n-1)(b+n)}\, z^n$$

is at least 1.

[01.9] From the very definition of convergence, show that when the partial sums of a series $a_1 + a_2 + \ldots$ are *bounded*, and when the elements of the sequence $\{b_n\}$ *are positive* (real) and *go to* 0 *monotonically*, then the series $\sum a_n b_n$ *converges*.

[01.10] Show that the function $f(z) = \sum z^n/n^2$ on the open disk $|z| < 1$ extends to a *continuous* function on the *closed* unit disc.
