(*November 5, 2014*)

Complex analysis examples 05

Paul Garrett garrett@math.umn.edu http://www.math.umn.edu/˜garrett/

[This document is http://www.math.umn.edu/˜garrett/m/complex/examples_2014-15/cx_ex_05.pdf]

[05.1] Compute $\displaystyle\int_0^\infty \frac{x^s\;dx}{x^2-x+1}$

[05.2] Compute $\displaystyle\int_{-\infty}^\infty e^{2\pi i\xi x}\;e^{-\pi x^2}\;dx$

[05.3] Compute $\displaystyle\int_{-\infty}^\infty \frac{\sin\xi x\;dx}{x(x^2+1)}$

[05.4] Show that an entire function taking values in the right half-plane $\mathrm{Re}(z)\geq 1$ must be constant.

[05.5] Adapt the reflection principle to show that a holomorphic function on the unit disk, extending to a continuous function on the closed unit disk, with $|f(z)|=1$ on the unit circle, extends to an entire function. (Hint: for example, $z\to\frac{z-i}{-iz+1}$ maps the real line to the unit circle.)
