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Complex analysis examples 07

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[This document is http://www.math.umn.edu/˜garrett/m/complex/examples_2014-15/cx_ex_07.pdf]

If you want feedback from me on your treatment of these examples, please get your work to me by Monday, Dec 01, preferably as a PDF emailed to me.

[07.1] Exhibit a linear fractional transformation mapping $1, 2, 3$ to z_1, z_2, z_3.

[07.2] Exhibit a linear fractional transformation mapping the circle $|z| = 1$ to the line $\mathrm{Re}(z) = \mathrm{Im}(z)$.

[07.3] Exhibit a linear fractional transformation stabilizing the (open) upper half-plane $\mathfrak{H}$ and mapping i to $2 + i$.

[07.4] Given $0 < t < 1$, exhibit a linear fractional transformation stabilizing the open unit disk, and mapping 0 to t.

[07.5] Exhibit a conformal map of the sector $\{re^{i\theta} : r > 0,\ 0 < \theta < \frac{\pi}{4}\}$ to the unit disk.

[07.6] Exhibit a conformal map from the strip $\{z = x + iy : c < ax + by < c'\}$ to the crescent

$$\Omega \;=\; \{z : |z| < 1,\ |z - \tfrac{1}{2}| > \tfrac{1}{2}\}$$

[07.7] Let holomorphic $f : \mathbb{CP}^1 \to \mathbb{CP}^1$ be 2-to-1. Show that there are two linear fractional transformations α, β such that $\alpha \circ f \circ \beta$ is the map $z \to z^2$.

[07.8] What happens to the zero set $\mathbb{Z}$ of $z \to e^{2\pi iz}$ under the perturbation $z \longrightarrow e^{2\pi iz} - hz$ for small h?
