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Complex analysis examples 11

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[This document is http://www.math.umn.edu/˜garrett/m/complex/examples_2014-15/cx_ex_11.pdf]

If you want feedback from me on your treatment of these examples, please get your work to me by Friday, Apr 24, preferably as a PDF emailed to me.

[11.1] Determine the genus of the curve $y^2 = x^5 - 1$.

[11.2] Show a change of variables to convert $y^2 = x^6 - 1$ to something of the form $y^2 =$ quintic in x.

[11.3] Determine the genus of the curve $y^3 = x^3 - 1$.

[11.4] Determine the genus of the curve $y^3 = x^4 - 1$.

[11.5] Determine the local ramification above $x = 0$ in the ramified cover $(x, y) \to x \in \mathbb{P}^1$ where $y^5 + xy^2 + x^2 = 0$.

[11.6] Determine the local ramification above $x = 0$ in the ramified cover $(x, y) \to x \in \mathbb{P}^1$ where $y^5 + x^2y^2 + x^2 = 0$.

[11.7] Show that a ramified cover $f : E_1 \to E_2$ of elliptic curves E_j must actually be *unramified*, that is, not ramified at any point.

[11.8] Show that in a ramified cover $C_1 \to C_2$ of compact connected Riemann surfaces, the genus of C_1 must be at least the genus of C_2.

[11.9] Determine the points z such that there is non-trivial ramification over z in the ramified covering $(z, w) \to z$ from the curve $w^5 + 5zw + z^3 = 0$.

[11.10] Let $z_1, \ldots, z_n$ be points in $\mathbb{P}^1$. Determine the dimension of the space of meromorphic functions on $\mathbb{P}^1$ with poles at most at $\{z_1, \ldots, z_n\}$, counting multiplicities.

[11.11] Let $\zeta_1, \ldots, \zeta_m$ and $z_1, \ldots, z_n$ be points in $\mathbb{P}^1$. Determine the dimension of the space of meromorphic functions on $\mathbb{P}^1$ with poles at most at $\{z_1, \ldots, z_n\}$, counting multiplicities, and zeros (at least) at $\zeta_1, \ldots, \zeta_m$.

[11.12] Let $z_1, \ldots, z_n$ be points on an elliptic curve $E = \mathbb{C}/\Lambda$. Determine the dimension of the space of meromorphic functions on E with poles at most at $\{z_1, \ldots, z_n\}$, counting multiplicities.
