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Complex analysis final exam Fall 2014

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[Fall 2014.1] Determine the Laurent expansion of $f(z) = 1/(z-1)(z-2)$ in the annulus $2 < |z|$.

[Fall 2014.2] Evaluate $\int_0^\infty \frac{\sqrt{x}\,dx}{x^2+x+1}$

[Fall 2014.3] Classify the holomorphic functions f on $\mathbb{C}$ satisfying $|f(z)| \le |z|^2$ for all $z \in \mathbb{C}$.

[Fall 2014.4] Show that there is a holomorphic function $f(z)$ on a neighborhood of 0 with $f(z)^2 = \frac{e^z-1}{z}$. Determine the radius of convergence.

[Fall 2014.5] Show that $f(z) = \sin z - z$ has at least two complex zeros.

[Fall 2014.6] Give an explicit conformal map of

$$\{z = x + iy : |z| < 1,\ x > -\tfrac{1}{2}\}$$

to the unit disk $|z| < 1$.
