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Complex analysis final exam Fall 2014-a

Paul Garrett garrett@math.umn.edu http://www.math.umn.edu/~garrett/

[Fall 2014.1] Determine the Laurent expansion of $f(z) = 1/z(z-1)$ in the annulus $1 < |z|$.

[Fall 2014.2] Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{x^2+1\;dx}{x^4+1}$

[Fall 2014.3] Classify the holomorphic functions f on the unit disk that extend to continuous functions on the *closed* unit disk, satisfying $|f(z)| = 1$ for all $|z| = 1$ and $f(z) \neq 0$ for $|z| \leq 1$.

[Fall 2014.4] Show that there is a holomorphic function $f(z)$ on a neighborhood of 0 with $\log f(z) = \frac{e^z-1}{z}$. Determine the radius of convergence.

[Fall 2014.5] Count the zeros of $z^6 - 3z^4 + 6z^2 - 1$ in $|z| < 1$.

[Fall 2014.6] Give an explicit conformal map of the slit disk

$$\{z = x + iy : |z| < 1, \text{ excluding } z \text{ with } y = 0 \text{ and } 0 \leq x < 1\}$$

to the disk without the slit, $|z| < 1$.
