

(November 1, 2014)

Complex analysis midterm 02

Paul Garrett garrett@math.umn.edu <http://www.math.umn.edu/~garrett/>

Please write on one side of a page, with your name on every page.

Please restate the respective questions, and respond in complete sentences, in standard English, legibly. The goal is *explanation* and also *persuasion*, not crypticness or telegraphic-ness.

Responses should be intelligible *without* definitive prior expertise. That is, the message(s) should be intelligible without knowing the message(s) in advance.

Questions are equally weighted.

[02.1] Compute $\int_{-\infty}^{\infty} e^{i\xi x} e^{-x^2} dx$ for real ξ .

[02.2] Compute $\int_0^{\infty} \frac{x^s dx}{x^2 - x + 1}$.

[02.3] Show that a holomorphic function f on a non-empty open set $U \subset \mathbb{C}$ such that $|f(z)| = 1$ for all $z \in U$ is necessarily constant.

[02.4] Show that there is a holomorphic $f(z) = \sqrt[3]{z^4 - 1}$ near any point z_0 with $z_0^4 \neq 1$. Determine the radius of convergence of the power series for $f(z)$ expanded at 0.
