Please write on one side of a page, with your name on every page.

Please restate the respective questions, and respond in complete sentences, in standard English, legibly. The goal is explanation and also persuasion, not crypticness or telegraphic-ness.

Responses should be intelligible without definitive prior expertise. That is, the message(s) should be intelligible without knowing the message(s) in advance.

Questions are equally weighted.

[03.1] Give an explicit conformal map of the half-disk \( \{ z = x + iy : |z| < 1, \ x > 0 \} \) to the unit disk \( \{ z : |z| < 1 \} \).

[03.2] Determine a finite set \( S \subset \mathbb{C} \) of points such that for \( w_o \notin S \) there is a holomorphic function \( f(w) \) near \( w_o \) such that \( z = f(w) \) gives a solution to the equation \( z^5 - 5z - w = 0 \). (Hint: holomorphic inverse function theorem.)

[03.3] Show that \( f(z) = e^{iz} - z \) has at least one complex zero.