

(March 2, 2015)

## Complex analysis midterm 04

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Please write on one side of a page, with your name on every page.

Please restate the respective questions, and respond in complete sentences, in standard English, legibly. The goal is *explanation* and also *persuasion*, not crypticness or telegraphic-ness.

Responses should be intelligible *without* definitive prior expertise. That is, the message(s) should be intelligible without knowing the message(s) in advance.

Questions are equally weighted.

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[04.1] Give a harmonic function  $u$  on the annulus  $\frac{1}{2} \leq u \leq 2$  such that on the *outer* boundary circle  $|z| = 2$  the boundary-value function is  $u(2e^{i\theta}) = e^{i\theta}$ , while on the *inner* boundary circle  $|z| = \frac{1}{2}$  the boundary-value function is  $u(\frac{1}{2}e^{i\theta}) = e^{-i\theta}$ .

[04.2] Show that for  $t \in \mathbb{R}$

$$|\Gamma(\frac{1}{2} + it)|^2 = \frac{2\pi}{e^{\pi t} + e^{-\pi t}}$$

[04.3] Prove that

$$\prod_{n \geq 3} \left(1 + \frac{1}{n \log n}\right) = +\infty \quad \text{and} \quad \prod_{n \geq 3} \left(1 - \frac{1}{n \log n}\right) = 0$$

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