Let $G$ be a finite abelian group, with dual group $\hat{G}$. Show that $\hat{G}$ separates points on $G$, in the sense that, given $x \neq y$ in $G$, there is a character $\chi \in \hat{G}$ such that $f(x) \neq f(y)$.

**Remark:** In the following, I had confounded two potentially correct assertions into a single incorrect one: representations induced from non-trivial characters on the center are not irreducible, but they are isotypic, meaning that they're direct sums of copies of a single representation.

Define a finite Heisenberg group by

$$H = \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} : a, b, c \in \mathbb{F}_q \right\}$$

with a fixed finite field $\mathbb{F}_q$. Verify that the center is

$$Z = \left\{ \begin{pmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : a, b, c \in \mathbb{F}_q \right\}$$

Verify that $H/Z \cong \mathbb{F}_q \oplus \mathbb{F}_q$, so is abelian. Thus, it is easy to identify many one-dimensional representations of $H$, by factoring through $H/Z$. The not-so-easy question is to show that, given a non-trivial central character $\omega : Z \to \mathbb{C}^\times$, there is a unique (isomorphism class of) irreducible representation(s) of $H$ with central character $\omega$. In particular,

$$\text{Ind}^H_Z \omega = \text{isotypic (not irreducible!)} \quad \text{for } \omega \neq 1$$

Given a non-trivial character $\omega$ on $Z$, extend it to $A$ by

$$A = \text{a maximal abelian subgroup} = \begin{pmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

by

$$\bar{\omega} : \begin{pmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mapsto \omega(b)$$

Then

$$\text{Ind}^H_A \bar{\omega} = \text{the unique irreducible with central character } \omega$$

**Hint:** It is possible to prove some of these things for finite Heisenberg groups by counting, if one knows that the (bi)regular representation of a finite (or even compact) group is the sum of all irreducibles, each occurring with multiplicity equal to its dimension. Use the elementary inequality

$$n_1^2 + \ldots + n_\ell^2 < (n_1 + \ldots + n_\ell)^2 \quad \text{for } n_i > 0, \text{ when } \ell > 1$$