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Modular forms and number theory exercises 14

Paul Garrett garrett@math.umn.edu http://www.math.umn.edu/~garrett/

[mfms 14.1] Show that

$$\exp\begin{pmatrix} 0 & t \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \qquad \exp\begin{pmatrix} s & 0 \\ 0 & t \end{pmatrix} = \begin{pmatrix} e^s & 0 \\ 0 & e^t \end{pmatrix}$$

$$\exp\begin{pmatrix} 0 & t \\ -t & 0 \end{pmatrix} = \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix} \qquad \exp\begin{pmatrix} 0 & t \\ t & 0 \end{pmatrix} = \begin{pmatrix} \cosh t & \sinh t \\ \sinh t & \cosh t \end{pmatrix}$$

$$\exp\begin{pmatrix} 0 & a & 0 \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & a & \frac{1}{2}ab \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}$$

[mfms 14.2] For a matrix A, prove that

$$\frac{d}{dt}e^{tA} = A \cdot e^{tA}$$

[mfms 14.3] For two matrix-valued functions A, B of real t, prove Leibniz' rule

$$(A \cdot B)' = A' \cdot B + A \cdot B'$$

[mfms 14.4] Prove that $\det(\exp A) = e^{\mathrm{tr}A}$ for matrices A, where tr is trace.

[mfms 14.5]* Give an example, with proof, of a real 2-by-2 matrix *not* hit by the exponential map from real 2-by-2 matrices.

[mfms 14.6]** Explain the exponential map for a Lie group that is not a *matrix* group.