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Number theory exercises 05

Paul Garrett garrett@math.umn.edu http://www.math.umn.edu/~garrett/

Due Wed, 16 Nov 2011, preferably as PDF emailed to me.

[number theory 05.1] Let $\mathfrak{o}$ be integrally closed in its quotient field k. Let K be a finite Galois extension of k, and $\mathcal{O}$ the integral closure of $\mathfrak{o}$ in K. For an intermediate field $k \subset E \subset K$, show that $E \cap \mathcal{O}$ is the integral closure of $\mathfrak{o}$ in E.

[number theory 05.2] Let p, q, r be distinct primes in $\mathbb{Z}$. Show that the ring $\mathfrak{o} = Z/pqr$ has exactly three prime ideals, generated by (the images of) p, q, r. Let $S = \mathfrak{o} - p\mathfrak{o}$, and compute the localization $S^{-1}\mathfrak{o}$.

[number theory 05.3] Let Φ_{15} be the 15^{th} cyclotomic polynomial

$$\Phi_{15}(x) \;=\; \frac{(x^{15}-1)(x-1)}{(x^3-1)(x^5-1)}$$

Show that, although Φ_{15} is irreducible in $\mathbb{Q}[x]$, it is *reducible* in $\mathbb{F}_p[x]$ for every prime p.
