Examples Diagnostic

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[This document is http://www.math.umn.edu/~garrett/m/real/examples_2018-19/Diagnostic.pdf]

[Diagnostic.1] What does it mean for a real-valued function on a subset Ω of $\mathbb{R}^n$ to be continuous?

[Diagnostic.2] What does it mean for a real-valued function on a subset Ω of $\mathbb{R}^n$ to be uniformly continuous?

[Diagnostic.3] What does it mean for a sequence $\{f_n\}$ of real-valued functions on a subset Ω of $\mathbb{R}^n$ to approach another $\mathbb{R}$-valued function on Ω pointwise?

[Diagnostic.4] What does it mean for a sequence $\{f_n\}$ of real-valued functions on a subset Ω of $\mathbb{R}^n$ to approach another $\mathbb{R}$-valued function on Ω uniformly (pointwise)?

[Diagnostic.5] For a sequence $\{f_n\}$ of continuous functions on an interval $[a, b]$, approaching a function $f$ uniformly pointwise, why is the limit $f$ continuous?

[Diagnostic.6] For a sequence $\{f_n\}$ of continuous functions on an interval $[a, b]$, approaching a function $f$ uniformly pointwise, why is $\lim_n \int_a^b f_n(x) \, dx = \int_a^b f(x) \, dx$?

[Diagnostic.7] Give several examples of sequences $\{f_n\}$ of continuous real-valued functions on an interval $[a, b]$, converging pointwise, such that $\lim_n \int_a^b f_n(x) \, dx \neq \int_a^b \lim_n f_n(x) \, dx$.

[Diagnostic.8] What does it mean for a subset of $\mathbb{R}^n$ to be compact?

[Diagnostic.9] Why is a continuous real-valued function on a compact subset of $\mathbb{R}^n$ uniformly continuous?

[Diagnostic.10] What is a metric space?

[Diagnostic.11] What does it mean for a real-valued function on a metric space to be continuous?

[Diagnostic.12] What does it mean for a real-valued function on a metric space to be uniformly continuous?

[Diagnostic.13] What does it mean for a metric space to be complete?

[Diagnostic.14] What does it mean for a subset of a metric space to be compact?

[Diagnostic.15] What does it mean that $\mathbb{Q}$ is dense in $\mathbb{R}$?

[Diagnostic.16] What does it mean that a subset $Y$ of a metric space $X$ is dense in $X$?

[Diagnostic.17] Why is $\mathbb{Q}$ countable? Why is $\mathbb{R}$ uncountable?