If you want feedback on your write-ups on any of these examples, please get your write-ups by Monday, 12 Sept, 2016.

[0.1] (There is not much hope in making sense of the outcome of an uncountable number of non-zero operations:) Let \( \Omega \) be an uncountable collection of positive real numbers. Letting \( F \) range over all finite subsets of \( \Omega \), show that \( \sup F \sum_{\alpha \in F} \alpha = +\infty \).

[0.2] (The archimedean property of the real numbers:) Using the characterization of \( \mathbb{R} \) as the metric-space completion of \( \mathbb{Q} \), show that a real number \( x \) with \( |x| < \frac{1}{n} \) for \( n = 1, 2, 3, \ldots \) must be 0.

[0.3] Prove carefully that the inf of a finite set of (strictly) positive real numbers is (strictly) positive.

[0.4] Prove (or review the proof) that intervals \([a, b] \subset \mathbb{R}\) (with \( -\infty < a < b < \infty \)) are connected in the sense that they cannot be written as a disjoint union of two non-empty (relatively) open subsets. Use this to prove the intermediate value theorem for continuous functions.

[0.5] Prove (or review the proof) that a continuous real-valued function \( f \) on a finite interval \([a, b] \subset \mathbb{R}\) assumes its inf. That is, there is a point \( x_0 \in [a, b] \) such that \( f(x_0) = \inf_{x \in [a, b]} f(x) \).

[0.6] Prove (or review the proof) that a continuous real-valued function \( f \) on a finite closed interval \([a, b] \subset \mathbb{R}\) is uniformly continuous: for all \( \varepsilon > 0 \) there is \( \delta > 0 \) such that, for all \( x, y \in [a, b] \), \( |x - y| < \delta \) implies \( |f(x) - f(y)| < \varepsilon \).