Solutions for Midterm 1

1. \( v(t) = 3 + 2t \) and \( x(t) = x_0 + 3t + t^2 \), so the distance traveled during the first \( t \) seconds is \( x(t) - x_0 = 3t + t^2 \). Setting \( t = 3 \) produces

   Answer: \( 3 \cdot 2 + 3^2 = 15 \text{ m} \).

2. Separating the variables produces \( \frac{dy}{y} = 3x^2 \, dx \). Integrating both sides we get \( \ln|y| = x^3 + C \). Exponentiating we get

   Answer: \( y = Ce^{x^3} \).

3. \( P(x) = 3 \). Integrating this function we get \( 3x \). Hence the integrating factor is \( \rho = e^{3x} \). Multiplying the equation by \( e^{3x} \) produces \( e^{3x}y' + 3e^{3x}y = 2e^{2x} \). The left hand side is \( (e^{3x}y)' \). Integrating both sides gives \( e^{3x}y = 2e^{2x} + C \). Setting \( x = 0 \) and \( y = 1 \) we get \( 1 = 1 + C \), i.e. \( C = 0 \). Dividing by \( e^{3x} \) we get

   Answer: \( y = e^{-x} \).

4. Separating the variables we get \( \frac{dx}{2x-x^2} = dt \). Using partial fractions we write \( \frac{dx}{2x-x^2} = \frac{1}{2} \left( \frac{1}{x-2} - \frac{1}{x} \right) \, dx \). Integration produces \( \ln|x-2| - \ln|x| = t + C \). Setting \( t = 0 \) and \( x = 1 \) we get \( C = 0 \). Since \( |x| = x \) and \( |x-2| = 2-x \) for the initial value of \( x \) (i.e. for \( x = 1 \)) we set \( \ln|x-2| = \ln(2-x) \) and \( \ln|x| = \ln x \). The equation now becomes \( \ln(2-x) - \ln(x) = t \), i.e. \( \ln \frac{2-x}{x} = t \). Exponentiating we get \( \frac{2-x}{x} = e^t \). Solving for \( x \) produces

   Answer: \( x(t) = \frac{2}{e^t+1} \).

5. We set \( x_0 = 0, y_0 = 1, x_1 = 0.2 \) and \( x_2 = 0.4 \); we need to compute \( y(0.4) = y(x_2) = y_2 \). First we compute \( y_1 = 1 + 0.2(2 \cdot 0 - 1^2) = 0.8 \) and then \( y_2 = 0.8 + 0.2(0.2 - 0.8^2) = 0.712 \).

   Answer: 0.712.

6. The augmented matrix is

   \[
   \begin{pmatrix}
   1 & 3 & 15 & 7 \\
   2 & 7 & 34 & 17 
   \end{pmatrix}
   \]

   and the matrix in echelon form is

   \[
   \begin{pmatrix}
   1 & 3 & 15 & 7 \\
   0 & 1 & 4 & 3 
   \end{pmatrix}
   \]

   Answer: infinitely many solutions; setting \( x_3 = t \), the parametric description of the solution set is \( (-2 - 3t, 3 - 4t, t) \).