Here is a summary of the most important parts of the first lecture.

- **Affine linear functions.** $y = mx + b$. If $b = 0$, the function is linear. Every affine linear function can be obtained by scaling and shifting the linear function $y = x$. (Scaling by a negative factor causes a reflection.)

- **Quadratic functions.** $y = ax^2 + bx + c$. Every quadratic function can be obtained by scaling and shifting the quadratic function $y = x^2$. This change of variables is closely related to “completing the square” and the quadratic formula.

- **Power functions** $y = ax^p$. Every power function can be transformed into an affine linear function by making a “log $x$, log $y$” change of variables.

- **Exponential functions.** $y = ab^cx$. The constant $b$ is called the “base”. We will primarily be interested in the case $b = e \approx 2.71829 \ldots$. Every exponential function can be obtained by scaling the exponential function $y = e^x$ (no shifting is required). Any exponential function can be transformed into an affine linear function by making an “$x$, log $y$” change of variables.

- **Logarithmic functions.** These are the inverses of the exponential functions, and they can all be transformed into the function $y = \log x$ by scaling.

- **Trigonometric functions.** $y = a \sin(bx + c), y = a \cos(bx + c), y = a \tan(bx + c)$. Any function of the form $y = a_1 \sin(bx + c_1) + a_2 \cos(bx + c_2)$ can be obtained by scaling and shifting the function $y = \sin x$.

- **Inverse trig functions.** The most important is $y = \arctan x$.

- **Other types of functions.** Higher degree polynomials and rational functions will occasionally be needed.

- **Functions obtained by combining other functions, especially by the composition of functions.** A good example is $y = \exp(-x^2/2)$, whose graph is a bell-shaped curve. It is the composition of the function $f(x) = e^x$ and the function $g(x) = -x^2/2$. This composition is written $f(g(x))$ or $f \circ g(x)$.

- **Differentiation.** You should review the product, quotient, power, and chain rules.
• **The Fundamental Theorem of Calculus.** This is the key connection between differentiation and integration.

• **Integration.** You should review basic properties, and the techniques of integration by parts and substitution. The most common usage of integration by parts involves integrals of the form \( \int xf(x) \, dx \), where \( f(x) \) is an exponential or trigonometric function. A common usage of substitution involves integrals of the form \( \int f(ax + b) \, dx \). This allows us to find integrals of functions that are obtained by scaling and shifting other functions. Another common usage of substitution involves integrals of the form \( \int xf(x^2) \, dx \), which is useful with a function like \( \exp(-x^2/2) \).

• **Linear combinations.** A linear combination of two variables \( x \) and \( y \) is \( c_1 x + c_2 y \), where \( c_1, c_2 \) are real numbers, called “coefficients”. A linear combination of two functions \( f(x) \) and \( g(x) \) is \( c_1 f(x) + c_2 g(x) \).

• **Linearity.** A function or operation is called linear if it “respects” linear combinations. For example, a function \( f \) is linear if \( f(c_1 x + c_2 y) = c_1 f(x) + c_2 f(y) \) for all coefficients \( c_1, c_2 \). Differentiation and integration are both linear operations: \((c_1 f(x) + c_2 f(x))’ = c_1 f’(x) + c_2 f’(x)\) and similarly for integration. Linearity is one of the most important concepts in the course, and we will talk about it every week!

• **Affine linear transformations of the \( xy \)-plane.** These can be best studied in the form

\[
\begin{pmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix}
+
\begin{pmatrix}
b_1 \\
b_2
\end{pmatrix}.
\]

By using matrices, we have a nice, clean, organized way to think about affine linear changes of variables, which includes scaling, shifting, rotations, reflections, and shearing. We will return to this topic in much more detail later on, so don’t worry too much about it for now.

As you review what you learned in the past about these topics, don’t hesitate to ask me questions about those items that give you difficulty. Next week, we will focus on getting a deep understanding of differentiation, integration, and how these two operations are useful.