1. (15 points) Find an equation for the plane passing through all three points \( \langle x, y, z \rangle = \langle 3, -2, -2 \rangle, \langle 2, 0, 1 \rangle \) and \( \langle 1, 0, 0 \rangle \).

**SOLUTION:** A normal vector \( \vec{v} \) is the cross product of \( \langle 3, -2, -2 \rangle - \langle 1, 0, 0 \rangle \) and \( \langle 2, 0, 1 \rangle - \langle 1, 0, 0 \rangle \). So

\[
\vec{v} = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
2 & -2 & -2 \\
1 & 0 & 1
\end{vmatrix} = -2\vec{i} - 4\vec{j} + 2\vec{k}.
\]

We can divide \( \vec{v} \) by \(-2\), so an equation for the plane is \( (x - 1) + 2(y - 0) - (z - 0) = 0 \), equivalently

\[x + 2y - z = 1.\]

2. (15 points) Find an equation for the surface in \((x, y, z)\)-space obtained by rotating the hyperbola \( x^2 - 4z^2 = 1 \) of the \((x, z)\)-plane about the \(x\)-axis.

**SOLUTION:** \( |z| \) is the distance form the \(x\)-axis in the \((x, z)\)-plane; we want to replace it with the distance to the \(x\)-axis in space, namely \( \sqrt{y^2 + z^2} \). The equation of the surface of revolution is

\[x^2 - 4y^2 - 4z^2 = 1.\]
3. (15 points) The lines given parametrically by
\[ (x, y, z) = (7 + 2t, -1 - t, -2t), \quad -\infty < t < \infty \]
and
\[ (x, y, z) = (4 - s, -1 + 2s, 2 + 2s), \quad -\infty < s < \infty \]
intersect at the point \( (x, y, z) = (3, 1, 4) \). Find an equation for the plane which contains both lines.

**SOLUTION:** A normal vector \( \vec{v} \) to the plane is the cross product of the vector multiplied by \( t \) in the first line and the vector multiplied by \( s \) in the other line:
\[
\vec{v} = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
2 & -1 & -2 \\
-1 & 2 & 2
\end{vmatrix} = -6\vec{i} - 6\vec{j} + 3\vec{k}.
\]
Divide \( \vec{v} \) by 3. So the plane is given by the equation
\[-2(x - 3) - 2(y - 1) + (z - 4) = 0, \quad \text{or} \quad -2x - 2y + z = -4.\]

4. (15 points) For the function \( f(x, y) = e^{-2y}\sin 2x \), find the **second partial derivatives**
\[ f_{xx} = \frac{\partial^2 f}{\partial x^2} \]
and
\[ f_{yy} = \frac{\partial^2 f}{\partial y^2}. \]

**SOLUTION:** \( f_x = 2e^{-2y}\cos 2x \), so \( f_{xx} = -4e^{-2y}\sin 2x \). For the \( y \) partial derivatives, \( f_y = -2e^{-2y}\sin 2x \) and \( f_{yy} = +4e^{-2y}\sin 2x \).

5. (10 points) Suppose \( z = f(x, y) \) is a function with partial derivatives \( f_x(3, 1) = 5 \) and \( f_y(3, 1) = 2 \). If \( x \) and \( y \) are both functions of \( t \): \( x = 5 - 2t \) and \( y = 2 + t - 2t^2 \), find
\[ \frac{dz}{dt} \]
at \( t = 1 \).

**SOLUTION:** \( x = g(t) = 5 - 2t \) so \( x = g(1) = 3 \) at \( t = 1 \), and \( \frac{d}{dt}x \, dt = g'(t) = -2 \).
Meanwhile, \( y = h(t) = 2 + t - 2t^2 \), so \( y = h(1) = 1 \) at \( t = 1 \). \( \frac{dy}{dt} = h'(t) = 1 - 2t \), so \( h'(1) = -1 \).
The **chain rule** says that
\[ \frac{dz}{dt} = f_x(3, 1)g'(1) + f_y(3, 1)h'(1) = (5)(-2) + (2)(-3) = -16. \]
6. (15 points) The point \( \langle x, y, z \rangle = \langle 2, 1, 0 \rangle \) lies on the surface \( S: \)
\[
x^2 - y^2 + xz + xy - 4z^2 = 5.
\]
Find the equation of the tangent plane to the surface \( S \) at \( \langle 2, 1, 0 \rangle \), in the form \( ax + by + cz = d \).

**SOLUTION:** The normal vector to the tangent plane to the surface \( g(x, y, z) = 0 \) is the gradient \( \vec{\nabla} g \). But \( g_x = 2x + z + y = 2 + 0 + 1 = 3 \); \( g_y = -2y + x = -2 + 2 = 0 \); and \( g_z = x - 8z = 2 - 0 = 2 \). So \( \vec{\nabla} g(2, 1, 0) = \langle 3, 0, 2 \rangle \). The equation of the tangent plane is
\[
3(x - 2) + 0(y - 1) + 2(z - 0) - 0 = 6.
\]

7. (15 points) (a) Find the gradient of the function \( f(x, y, z) = e^z \ln(x + 2y) \) at the point \( \langle x, y, z \rangle = \langle e, 0, 1 \rangle \). (b) Find the directional derivative of \( f \) at the point \( \langle e, 0, 1 \rangle \) in the direction
\[
\vec{u} = \frac{1}{3} (\vec{i} - 2\vec{j} + 2\vec{k}).
\]

(15 points) **SOLUTION:** \( f_x = \frac{e^z}{x+2y} = 1 \); \( f_y = 2f \frac{e^z}{x+2y} = 2 \); and \( f_z = e^z \ln(x+2y) = e \). So the gradient is
\[
\vec{\nabla} f(e, 0, 1) = \vec{i} + 2\vec{j} + e\vec{k}.
\]