We want to find the general solution of
\[ y'' + 2y' + y = 0 \]
in a way that explains “multiply by \( t \).” We make the substitution
\[ y = e^{-t}u. \]
We then have
\[
(e^{-t}u)'' + 2(e^{-t}u)' + e^{-t}u = e^{-t}((u - 2u' + u'') + (-2u + 2u') + u) = e^{-t}u''.
\]
Thus the diff. eqn. for \( u \) implied by the given diff. eqn. for \( y \) is
\[ e^{-t}u'' = 0 \]
or more simply
\[ u'' = 0 \]
after cancelling the factor of \( e^{-t} \). Performing indefinite integration twice we have
\[ u = C_1 + C_2t \]
and hence after reversing the substitution we get the previously mysterious-seeming formula
\[ y = C_1 e^{-t} + C_2 te^{-t} \]
for the general solution.