TYPO: I initially wrote the matrix $A$ on the blackboard with +5 in the lower left corner because, alas, I had written that in my lectures notes. (I had miscopied from a computer screen.) Later in the lecture I corrected the sign on the basis of circumstantial evidence provided by later matrices I wrote down. But then I felt obligated after class to run all the calculations over again and type out these notes using the computer to cut-and-paste in order to make sure I had not introduced any further error. Indeed I hadn’t.

$$A = \begin{bmatrix} -17 & 40 & 20 \\ -5 & 13 & 5 \\ -5 & 10 & 8 \end{bmatrix}, \ 
\lambda = -2, 3, 3$$

$$A - (-2)I_3 = \begin{bmatrix} -15 & 40 & 20 \\ -5 & 15 & 5 \\ -5 & 10 & 10 \end{bmatrix}, \ \text{rref}(A - (-2)I_3) = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 4 \\ 1 \end{bmatrix} \leftrightarrow -2$$

$$A - 3I_3 = \begin{bmatrix} -20 & 40 & 20 \\ -5 & 10 & 5 \\ -5 & 10 & 5 \end{bmatrix}, \ \text{rref}(A - 3I_3) = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2s + t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \leftrightarrow 3$$