Math 2373, CSE Lin. Alg. and Diff. Eq., Spring 2019, Univ. of Minn.

HOMEWORK ASSIGNMENTS THROUGH WEEK 12
(This is the complete listing of all homework assignments for the semester.)

Last update: Friday, April 12, 2019.

All assignments are from the required text by Farlow, Hall, McDill and West (except for a few “home-made” problems written right on this sheet). Answers to all problems must be justified—unexplained numerical answers will get no credit. Calculations must be done by hand unless you are given specific instructions to do otherwise. Remember that you are practicing up for the exams.

Week 1 homework due Tuesday, Jan. 29

• Sec. 3.2, pp. 143–145: 5,6,8,12,13,14,16,19,21,22,26,30,33,34
• Use the row operation notation on p. 134 of the text correctly to explain each step of row-reduction needed to reduce the matrices in problems 12,13,14,16,19,21,22 to reduced row echelon form. In 33,34 we give you a pass to use the \texttt{rref} command on your calculator, but other that that all work must be done by hand. Furthermore, in answering 26,30,33,34 don’t just discuss solutions—find all of them. In the case of more than one solution give the answer in the form demonstrated in Ex. 7 on pp. 139–140.

Week 2 homework due Tuesday, Feb. 5 “Weather Emergency Week”

• Sec. 3.1, pp. 127–130: 2,3,4,5,6,7,12,14,18,22
• Sec. 3.2, pp. 143–145: 1,2,4,66 (To do 66 you may use your calculator to calculate \texttt{rref} but then in case of infinitely many solutions you must write out the solution following Ex. 7 on pp. 139–140.)
• Sec. 3.4, pp. 164–167: 1,2,3,4,12,13,16,17,18,39,42 (When/if using row operations, use the notation from p.134 of the text. In 39 and 42, write out the determinants carefully but then you can evaluate them with your calculator.)
• Problem 15 from Sec. 3.4 is not assigned but you must know what it says to do 16, 17, 18.
• Sec. 2.2, pp. 70–73: 1,2,6,8,16,18: This portion “frozen out,” i.e., cancelled. (We use only the integrating factor method to solve first order linear differential equations in this course.)

Week 3 homework due Tuesday, Feb. 12

• Sec. 2.2, pp. 70–73: 1,2,6,8,16,18,22,23,29,30 (We use only the integrating factor method to solve first order linear differential equations in this course.)
• Sec. 3.3, pp. 154–156: 1,2,6,7,8,13,20,21 (Problems 20 and 21 use the important formula $x = A^{-1}b$ which needless to say you must know.)
• Sec. 3.3: 15 is not assigned but you must know it!
• Sec. 2.4, pp. 84–87: 2(a,c),3(a,c),6
• Home-made: An enormous tank initially contains 400 gallons of water in which is dissolved 50 lbs of salt. Brine containing 4 lbs of salt per gallon enters the tank at the rate of 5 gallons per minute. The well mixed brine leaves the tank at the slower rate of 3 gallons per minute. Find an expression for the number of pounds of salt in the tank at time $t$. (Don’t worry about the tank overflowing.)
Week 4 homework due Tuesday, Feb. 19

- Sec. 1.2, pp. 20–24: 1, 2, 13, 14, 16–21
- Sec. 1.3, pp. 29–32: 11, 14, 18, 25–30, 32
- Sec. 2.3, pp. 77–80: 4, 5, 7, 8, 12, 16, 24, 31
- Sec. 2.4, pp. 84–87: 16, 20
- Home-made problem 1: Solve the boundary value problem
  \[
  \frac{dy}{dt} = 0.06y - 12M, \quad y(0) = 250000, \quad y(30) = 0
  \]
  where as part of the problem you have to determine the constant $M$. Interpret your result in terms of paying down a mortgage.
- Home-made problem 2: Solve the initial value problem
  \[
  \frac{dy}{dt} = (y + 1)(y - 3), \quad y(0) = 1
  \]
  and sketch the solution on the same axes with the direction field of the differential equation and the equilibrium solutions. Say which equilibrium solution is stable and which unstable.
  *Typos corrected on February 10, 2019 in both home-made problems.*

Week 5 homework due Tuesday, Feb. 26

Midterm I, Feb. 19

- Sec. 4.1, pp. 205–210: 2, 3, 4, 15, 16, 17, 24, 25, 26, 40, 41, 42, 43
- Your answers to 40, 41, 42, 43 should each be justified by relating numerical details of the IVP to features of the graphs using new vocabulary in Sec. 4.1, e.g., period, circular frequency, and so on.

Week 6 homework due Tuesday, Mar. 5

- Sec. 3.6: p. 191: 7–11, 64
- You may use the rref button under the “full disclosure” rule for your work in Sec. 3.6 but you must still explain in words how you get your final answers.
- Sec. 4.2: pp. 222–229: 1, 2, 3, 5, 7, 11, 16, 17, 19, 22
- Sec. 4.3: pp. 238–243: 1, 2, 3, 7, 11, 12, 13, 16, 62
- Ignore the instruction “give a basis...” in 4.3: 1, 2, 3, 7
- Concepts from Sec. 3.6 are covered this week not only in homework and worksheets but also in the lab. You are responsible on upcoming tests for all of this material exclusive of that specifically related to the MATLAB software.

Week 7 homework due Tuesday, Mar. 12

- Sec. 3.4, pp. 164–167: 46, 50
- The method of problem 50 is needed to do one of the lab exercises.
- Sec. 4.4, pp. 253–254: 9, 13, 21, 23, 41, 43, 44, 46, 49
- Sec. 5.3, pp. 324–326: 2, 4, 7, 8, 11, 20, 22, 26, 30
- Ignore instruction to sketch eigenspaces. In case of complex eigenvalues write $A = \alpha I + \beta J$ where $\alpha \pm \beta i$ are the eigenvalues of $A$ and $\beta > 0$, as discussed in class. Skip the eigenvectors.
- For the three-by-three problems in §5.3 here are the eigenvalues:
  - Eigenvalues for 20 are 1, 2, 3.
  - Eigenvalues for 22 are −1, 0, 0.
  - Eigenvalues for 26 are −1, 2, 4.
  - Eigenvalues for 30 are 1, 1, 2.
- In general our policy is to supply eigenvalues for three-by-three matrices; we do not ask students to calculate them.
Week 8 homework due Tuesday, Mar. 26

- Sec. 4.6, pp. 270–273: 2,3,6,12,20,21,34(a,b,c,e)
- Problem 34 from Sec. 4.6 uses important vocabulary you must know.
- Sec. 5.4, pp. 338–341: 27,29,31,33,35,37,38,39,42,44
  (corrected March 12, 2019; see below for explanation)
- Hints:
  - Eigs. for prob. 35 are 0,1,2.
  - Eigs. for prob. 37 are 2,2,3. (The eigenvalue 2 is double.)
  - Eigs. for prob. 38 are 3,3,5. (The eigenvalue 3 is double.)
  - Eigs. for prob. 39 are -4,2,2. (The eigenvalue 2 is double.)
  - Eigs. for prob. 42 are -1,1,5.
  - Eigs. for prob. 44 are 1,2,3.
  - Note posted March 12, 2019: The three-by-three matrices for which
    I gave the eigenvalues are the ones I intended to assign. The problem list
    above previously did not quite match; it has been revised to match exactly.
- Problems 49,50 from Sec. 5.4 are not assigned but you must know what they say.
- Home-made 1: Find matrix powers $A^n$ of the following matrices $A$ copied from the previous
  part of the homework assignment so that you already know the eigenvalues and eigenvectors:
  $\begin{bmatrix} 12 & -6 \\ 15 & -7 \end{bmatrix}$,
  $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$
- Home-made 2: Find the points on the ellipse $8x^2 - 4xy + 5y^2 = 180$ nearest and farthest from
  the origin. Get started by rewriting the equation for the ellipse as
  $\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 8 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 180$.

Week 9 homework due Tuesday, Apr. 2

MT II: Tues., Mar. 26, 2019

- Sec. 6.2, pp.369–370: 9,10,26,28,40,42
  - Eigenvalues for problem 40 are 1,2,3
  - Eigenvalues for problem 42 are 2,0,2
  - Ignore the instructions to sketch solutions.
- Sec. 4.7, p. 282: 31 (This is setup only, no solving.)
- Home-made similar to Sec. 4.7: 31:
  - Convert each of the following to a system:
    - $y'' + 4y' + 4y = 3te^{-2t}$, $y(0) = 5$, $y'(0) = 2$
    - $y'' + 3y' + 2y = t^2 + 1$, $y(0) = 2$, $y'(0) = -4$
    - $2y'' + 10y' + 12y = 4t\sin(3t)$, $y(0) = 1$, $y'(0) = -2$
  - You do not have to solve the IVP’s or the systems.
  - Use the formula on p. 202 but use matrix notation and letter choices as explained in class.
  - Example: $3y'' + 6y' + 15y = 21\cos(t)$, $y(0) = -3$, $y'(0) = 1$ converts to
    $\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 7\cos(t) \end{bmatrix}$, $\begin{bmatrix} u(0) \\ v(0) \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ where $\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} y \\ y' \end{bmatrix}$
Week 10 homework due Tuesday, Apr. 9

- Sec. 1.4, pp. 43–45: 2
- §6.5, pp. 401: 15, 16, 17, 18
- For the problems assigned in §6.5 use undetermined coefficients.
- Eigenvalues for 17 are 1,1,3.
- Eigenvalues for 18 are 1,2,3.
- §6.7, pp. 418–419: 4,5,6, 17(a,b)
- For problems 4,5,6 assigned in §6.7 use undetermined coefficients. Problem 17(a,b) does not involve any solving, only setup and reconciling an already-worked-out answer with your setup.
- §8.1, pp. 474–475: 8,9,14,15,18,42,48,52
- For problems 8,9 in §8.1 you can use the table of indefinite integrals at the end of the textbook but not the table of Laplace transforms. For the rest of the problems you may use the table of Laplace transforms.
- §8.2: pp. 483–484: 5,6,9

Week 11 homework due Tuesday, Apr. 16

- §8.2: pp. 483–484: 7,8
- Home-made 1: Solve
  \[ y'' + 4y' + 13y = 44\sin(3t) + 132\cos(3t), \quad y(0) = 4, \quad y'(0) = 10 \]
  with \( \mathcal{L} \).
- Update Friday, April 12, 2019: After somebody noticed \( y(0) \) repeated twice I added a ‘‘prime’’ to the initial conditions above.
- §8.3: pp. 496–500: 1,2,4,10,11,12
- §8.3: pp. 496–500: 20,21,22,23,26
- For 20,21,22,23,26 besides following textbook instructions, you are instructed to rewrite each of the given functions as a “function-in-pieces” and to sketch its graph.
- §8.3: 30,31,32,36
- For 32 also provide a graph of the inverse Laplace transform.
- Home-made 2: Find the inverse Laplace transform of \( \frac{e^{-5s}}{s^2+4s+13} \).

Week 12 homework due Tuesday, Apr. 23

LAST HOMEWORK TO BE COLLECTED THIS SEMESTER

- §6.3, pp. 381: 13,15,16
- Use Laplace transforms efficiently as explained in class to solve these systems and point out where the matrix \( J \) appears in the Laplace transform calculation.
- §8.4, p. 507: 5,6,7,9
- Use Laplace transforms to calculate the convolutions, NOT the integral definition.
- Home-made problem 1: Use Laplace transforms to solve the following:
  \[ y'' + 4y' + 29y = 9\delta(t - 1), \quad y(0) = 0, \quad y'(0) = 1. \]
- Home-made problem 2: Use Laplace transforms to solve the following:
  \[ y'' + 2y' + 5y = 50H(t - 3), \quad y(0) = -1, \quad y'(0) = 7. \]
- Home-made problem 3: Find the convolution \( f(t) * g(t) \) where
  \[ f(t) = H(t - \pi) - H(t - 2\pi), \quad g(t) = \sin(t) \]
  by using Laplace transforms and graph this function.