This list of problems is not guaranteed to be an absolutely complete review for MT3. For completeness you must also make sure that you know how to do all of the homework assigned in the course up to and including that due on Tues., Nov. 26, and all the worksheet problems through week 12, with strong emphasis on weeks 9–12 but with the following exceptions:

- You will not have to compute Laplace transforms from the definition—you may in all cases use the table.
- You will not have to compute convolutions from the definition—in all cases you may use Laplace transforms.

Furthermore:

- New material on the worksheets for Week 13 (Thanksgiving week) will not be covered on MT3 but will be covered on the final exam.
- Review for the final exam starts on Wednesday of Week 14. The review includes some review of new material from Week 13.

An answer key for this review sheet will be posted on or before late Friday afternoon November 22. The exact coverage for the final exam will be addressed at a later date, shortly after MT3. The table of Laplace transforms supplied with this review sheet is the same as will be supplied on MT3 and on the final. Note that Appendix PF of the textbook provides help with partial fractions.

To study under typical test conditions, here are the ground rules. You may use matrix addition, inversion and multiplication functions on your calculator along with det and rref functions, but you must indicate clearly where and how you used these calculator functions—otherwise your answers would be considered unjustified and get little credit. Other more sophisticated calculator functions, e.g., those for finding eigenvalues and eigenvectors, are not allowed to justify your work. All algebra and calculus work (except for the matrix functions specifically allowed above and the typical scientific calculator functions +, −, ×, ÷, √, log, etc.) must be performed by hand and shown clearly. In questions involving matrices with variable entries you have to do all the multiplying by hand; only matrices with numerical entries may be multiplied by your calculator.
Problem 1

Solve the following IVP’s:

\[
\begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix} = \begin{bmatrix}
-7 & 2 & 0 \\
-6 & 0 & 0 \\
-29 & 20 & 14
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}, \quad \begin{bmatrix}
x(0) \\
y(0) \\
z(0)
\end{bmatrix} = \begin{bmatrix}
1 \\
1 \\
2
\end{bmatrix}
\]

To solve the second IVP use Laplace transforms because the eigenvalues are complex. For the last IVP we tell you that the eigenvalues are \(-12, 6, 12\).

Problem 2

Solve the following inhomogeneous IVPs:

\[
\begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix} = \begin{bmatrix}
-7 & 2 & 0 \\
-4 & -1 & 0 \\
14 & -3 & -5
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z
\end{bmatrix} + \begin{bmatrix}
63 \\
-126 \\
2
\end{bmatrix}, \quad \begin{bmatrix}
x(0) \\
y(0) \\
z(0)
\end{bmatrix} = \begin{bmatrix}
7 \\
-17 \\
2
\end{bmatrix}
\]

For the second problem the eigenvalues are \(-1, 1, 2\).

Problem 3

Find the Laplace transform of

\[
f(t) = \begin{cases} 
2t - 1 & \text{for } 0 \leq t < 2, \\
t + 7 & \text{for } 2 \leq t < 4, \\
8 & \text{for } t \geq 4. 
\end{cases}
\]

Problem 4

Find \(L\{(t + 1)^2e^{-3t}\}\) using the table of Laplace transforms supplied with this review sheet.

Problem 5

Solve the IVP

\[y'' + 4y' + 13y = -162e^{-5t}, \quad y(0) = 0, \quad y'(0) = 37\]

using Laplace transforms.

Problem 6

Find the inverse Laplace transforms of

\[
e^{-5s} \frac{5s - 9}{s^2 + 14s + 53}, \quad e^{-7s} \frac{2s + 7}{(s + 1)^2(s + 2)}, \quad e^{5s} \frac{5s - 9}{s^2 + 14s + 53}.
\]
Problem 7

Convert the IVP
\[ 2y'' + 20y' + 58y = -200 \cos(7t) - 700 \sin(7t), \quad y(0) = 2, \quad y'(0) = 31 \]
to an IVP for a system of first order equations.

Problem 8

Find the inverse Laplace transform \( f(t) \) of
\[ F(s) = e^{-s}/s + 2e^{-2s}/s^2 - 5e^{-3s}/s - 4e^{-5s}/s^2, \]
rewrite \( f(t) \) in piecewise form and sketch its graph.

Problem 9

We consider two brine tanks. Initially:
- Tank A contains 150 gallons of water and 37 pounds of salt.
- Tank B contains 250 gallons of water and 43 pounds of salt.

Starting at time \( t = 0 \):
- Brine at a concentration \( 3 \text{ lb gal}^{-1} \) of salt is pumped at \( 5 \text{ gal min}^{-1} \) into tank A.
- Brine is pumped from tank A to tank B at a rate of \( 5 \text{ gal min}^{-1} \).
- Brine at a concentration \( 2 \text{ lb gal}^{-1} \) of salt is pumped at \( 7 \text{ gal min}^{-1} \) into tank B.
- Brine is dumped down the drain from tank B at \( 12 \text{ gal min}^{-1} \).

Note that the amount of water in each tank remains constant. The brine is kept well-mixed at all times. Let \( x \) and \( y \) denote the amounts of salt in tanks A and B, respectively. Write down the IVP determining \( x \) and \( y \) for all times \( t \). You do not have to solve the IVP.

Problem 10

Solve the following IVP using Laplace transforms:
\[ y'' + 10y' + 41y = 164H(t - 3), \quad y(0) = 4, \quad y'(0) = -6. \]

Problem 11

Solve the IVP with \( L \):
\[ x'' + x = \delta(t - \pi) - 2\delta(t - 5\pi), \quad x(0) = 0, \quad x'(0) = 1. \]
Graph the solution for \( 0 \leq t \leq 8\pi \).

Problem 12

Use Laplace transforms to compute the convolution \( e^{2t} * t \).
### Table of Laplace transforms

<table>
<thead>
<tr>
<th>$f(t)$</th>
<th>$\mathcal{L}{f(t)} = F(s) = \int_0^\infty e^{-st} f(t) , dt$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a f(t) + b g(t)$</td>
<td>$a\mathcal{L}{f(t)} + b\mathcal{L}{g(t)} = aF(s) + bG(s)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$f(t)$</th>
<th>$L{f(t)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$1/s$</td>
</tr>
<tr>
<td>$t$</td>
<td>$1/s^2$</td>
</tr>
<tr>
<td>$t^2$</td>
<td>$2/s^3$</td>
</tr>
<tr>
<td>$t^n$</td>
<td>$n!/s^{n+1}$</td>
</tr>
<tr>
<td>$e^{at}$</td>
<td>$1/(s - a)$</td>
</tr>
<tr>
<td>$te^{at}$</td>
<td>$1/(s - a)^2$</td>
</tr>
<tr>
<td>$t^n e^{at}$</td>
<td>$n!/s^{n+1}$</td>
</tr>
<tr>
<td>$\sin(bt)$</td>
<td>$b/(s^2 + b^2)$</td>
</tr>
<tr>
<td>$\cos(bt)$</td>
<td>$s/(s^2 + b^2)$</td>
</tr>
<tr>
<td>$e^{at}\sin(bt)$</td>
<td>$b/(s - a)^2 + b^2$</td>
</tr>
<tr>
<td>$e^{at}\cos(bt)$</td>
<td>$(s - a)/(s - a)^2 + b^2$</td>
</tr>
<tr>
<td>$tf(t)$</td>
<td>$-\frac{d}{ds} F(s)$</td>
</tr>
<tr>
<td>$e^{at}f(t)$</td>
<td>$F(s - a)$</td>
</tr>
<tr>
<td>$f'(t)$</td>
<td>$sF(s) - f(0)$</td>
</tr>
<tr>
<td>$f''(t)$</td>
<td>$s^2F(s) - sf(0) - f'(0)$</td>
</tr>
<tr>
<td>$\int_0^t f(w)dw$</td>
<td>$F(s)/s$</td>
</tr>
<tr>
<td>$f(t) * g(t)$</td>
<td>$\int_0^t f(t - w)g(w)dw$</td>
</tr>
<tr>
<td>$H(t-a)$</td>
<td>$e^{-as}/s$</td>
</tr>
<tr>
<td>$H(t-a)f(t)$</td>
<td>$e^{-as}/s^2$</td>
</tr>
<tr>
<td>$H(t-a)f(t-a)$</td>
<td>$e^{-as}F(s)$</td>
</tr>
<tr>
<td>$H(t-a)f(t+a)$</td>
<td>$e^{-as}\mathcal{L}{f(t+a)}$</td>
</tr>
</tbody>
</table>

Note: We write $H(t) = \text{step}(t)$ to abbreviate. ($H$ is for Heaviside.)