Assignments are finalized for a given week by 5pm on Mondays.
All exercises come from the text by Edwards-Penney-Calvis unless explicitly marked “home-made” and written out in this document.
Only the beginning page of a sequence of exercises is given.

Week 1, Jan 16–19 [hmwk due Jan 23].

• Sections covered:
  – Wed: 3.1: systems of linear equations, “three possibilities,” method of elimination,
  – Fri: 3.2: coefficient matrices, augmented matrices, Gaussian elimination, row operations and notation for them, echelon form

• Homework assignment:
  – 3.1, p. 145: 3,5,7,9,11,15,19,23 (Each of the “three possibilities” appear in this list of homework problems. As part of your final answer to each problem say in a word or two which of the three cases holds.)
  – 3.1 (continued): 23 (Ignore the diff. eq. and just find the constants.)
  – 3.2, p. 154: 1,3,8 (Use vector notation for final answers as shown in class—this is much easier for us to read and you may be asked to use this notation on tests.)
  – 3.2 (continued): 13,15,20, 21 (Use the row operation notation on p. 149 to explain each step—you may be asked to use this notation on tests. Also report final answers in vector notation.)

• Remark: Practicing on more of the linear system problems than just the assigned ones is a good idea. Checking your answers with your calculator or Wolfram Alpha or some equivalent tool is easy and encouraged. Of course we want to see the steps on your homework, not just the final answer. The steps are what you get credit for and get tested on.

Date: Draft of Sunday, April 15, 2018.
Week 2, Jan 22–26 [hmwk due Jan 30]

- Sections covered:
  - Mon: 3.3: rref and also 1.4: separable diff. eqs.
  - Note added Jan. 22: For background on and vocabulary for differential equations also see section 1.1.
  - Wed: 3.4: Matrix operations and 1.4: population growth etc.
  - Fri: 3.5: Matrix inverse, \( x = A^{-1}b \) and 1.4: heating and cooling

- Homework assignment:
  - 1.4, p. 41: 6,17,19,23 (solve separable DEs)
  - 1.4: Home-made: Solve \( \frac{dy}{dt} = y(4 - y), y(0) = 3. \)
    (Home-made added Jan. 22.)
  - 1.4, p. 41: 34,35,38 (population growth)
  - 1.4, p. 41: 43,49,65 (heating/cooling)
  - 3.3, p. 162: 4,12,13,19
  - Each row operation must be labeled using notation on p. 149 of the text.
  - 3.4, p. 173: 3,8,9,13,15
  - 3.5, p. 185: 6,7
  - 3.5, p. 185: 16,17,19 (Again, label row operations.)
  - 3.5, p. 185: 23,24
  - As per usual, more practice on similar unassigned problems is good.
  - You are expected to calculate by hand on this assignment.

Week 3 (Jan 29–Feb 2) [hmwk due Feb 6]

- Sections covered:
  - Mon: 3.6(determinants: introduction; Cramer’s Rule), 1.5(first order linear DEs: introduction; integrating factor method)
  - Wed: 1.5 (mixture problems), 3.6(vocabulary, properties of determinants, computations)
  - Fri: 1.5 (more mixture problems), 3.6(transpose, adjoint formula for inverse)

- Homework assignment:
  - 1.5, p. 53: 2,3,6,13,14,17,33,36,37,41
  - 3.6, p. 201: 2,3,6,7,9,12,15,20,21,22,23,31,33,35

Week 4 (Feb 5–9) [hmwk due Feb 13]

- Sections covered:
  - Mon: 4.1
  - Wed: 4.2, 5.1
  - Fri: 5.1,5.2

- Homework assignment:
  - 4.1, p. 220: 2,3,6,7,11,12,15,18,19,20,25,28
  - 4.2, p. 227: 15,16,20
  - 5.1, p. 276: 2,3,6,7,9,12,34,35,36,39
  - 5.2, p. 288: 2,4,23,24
  - Whether a problem is odd or even, you have to show all the steps. The steps are what you get credit for. Linear algebra problems have to be solved by hand until after the first midterm.
Week 5 (Feb 12–16) [hmwk due Feb 20]

- Midterm I: Friday, February 16, 2018
- Sections (selectively) covered:
  - Mon: 5.3, 5.4: IVPs for homogeneous 2nd order linear DEs with constant coeffs.; free undamped motion; mass-spring-dashpot systems;
  - Wed: 5.3, 5.5: free damped motion; intro to “undetermined coefficients”; free fall with air resistance
- Homework assignment:
  - 5.3, p. 300: 21,22,23
  - 5.4, p. 311: 2,4,14(ignore the “show that...” instruction),18(ignore the “construct a figure...” instruction),22
  - 5.5, p. 325: 1,2,3,31,32,33
  - Home-made: Solve the initial value problem
    \[ y'' + 0.2y' = -32, \quad y(0) = 10000, \quad y'(0) = 0. \]
    Use the method of undetermined coefficients. Evaluate \( y \) at the time when \( y' \) is 90% of its limiting value as \( t \to +\infty \).

Week 6 (Feb. 19–23) [hmwk. due Feb 27]

- Sections covered:
  - Mon: Selective coverage of 4.4 plus “side-by-side” method (see example on web page). Further coverage of 5.3 and 5.5.
  - Wed: Further coverage of 5.3 and 5.5, with supporting coverage of complex numbers and Euler formula. Undetermined coefficients at “black belt” level. Further coverage of 4.4: formal definition of a basis, and methods for finding such. Quick intro. to 5.6
  - Fri: Continue 5.6 including practical resonance. Roots, powers, and logarithms of complex numbers (some material not in text).
- Homework assignment:
  - Home-made (practice with complex numbers). Find the square roots and cube roots of \( 3 - 4i \). Express final answers in \( a + bi \) form accurate to 4 places after the decimal. (Your calculator can do this but you have to explain your answer in terms of Euler’s formula and the laws of exponents.)
  - 4.4, p. 241: 7,8,9,10,11 (Just use the algorithm on p. 240 to produce a basis, i.e., color inside the lines.)
  - 4.4, p. 241: 21,26 (Follow p. 240 to get an answer, but also use the “side-by-side” method to get another answer, and reconcile the two answers. See note on the web page posted Feb. 19, 2018 for some guidance.)
  - 5.3, p. 300: 11,24,27,39,40,41,42
  - 5.3, p. 300: 43 is NOT assigned but you must know what 43(a) says.
  - 5.5, p. 325: 17,21,23,26,34
  - 5.6, p. 335: 7,9,17
- From now on (with few exceptions) you will not have to do row operations by hand. You can use the \texttt{rref} button under the “full disclosure” rule. You will have many systems of linear equations to solve in your study of differential equations. Get good at using \texttt{rref}.
Week 7 (Feb 26–Mar 2) [hmwk. due Mar 6]

• Sections covered:
  – Mon: 7.1: intro to systems of first order differential equations, conversion to first order systems; 6.1: intro to eigenvalues and eigenvectors
  – Wed: 7.2: linear systems of first order differential equations; Wronskians; 7.3: intro to eigenvalue method; 6.1: continuation as needed
  – Fri: 7.3: continuation; cascading brine tanks; 6.1: continuation as needed
  – Only real eigenvalues are considered this week.

• Homework assignment:
  – 6.1, p. 346: 1,5,9,12,15,17,22
  – 7.1, p. 371: 1,3,6,9
  – 7.2, p. 384: 2,11,12,14,15,24,25
  – 7.3, p. 395: 2,3,6,7,22,23,27,28,38
  – Note added Friday, March 2, 2018: Ignore the instruction to draw direction fields and graphs with a computer in problems 2,3,6,7

Week 8 (Mar 5-9) [hmwk due Mar 20, after spring break]

• Sections covered:
  – Mon: 7.3 again: two-by-two with complex eigenvalues; 6.2: intro. to diagonalization
  – Wed: 6.2: diagonalization (continued), Cayley-Hamilton theorem
    6.3: application to powers of matrices; “weather in Oz”
  – Fri: 8.1: matrix exponentials, mostly the two-by-two case

• Homework assignment:
  – 6.2, p. 353: 2,3,4,10,15,16,20,23
  – 6.3, p. 363: 2,4,10,12,19,20,21,26,27,30
  – In problems 19,20,21, just find $A^{-1}$ using Cayley-Hamilton. Note added March 7, 2018: I revised the previously mis-typed list of problems from 6.3.
  – 7.3, p. 395: 11,12,13,14 (ignore instruction to make graphs)
  – 8.1, p. 479: 2,3,5,6 (In these problems instead of finding a fundamental matrix as instructed, find the matrix exponential by the easy method discussed in class and then solve the IVP using it.)
  – Note added March 7, 2018: I added one further problem to the list from section 8.1.

Spring Break (Mar 12–16) yay!
Week 9 (Mar 19–23) [hmwk due Mar 27].

• **Midterm II, Friday, March 23, 2018**
  • Sections covered:
    – Mon: 10.1: $\mathcal{L}$ intro.; 10.3: $\mathcal{L}^{-1}$ and partial fractions intro.
    – Wed: 10.2 intro. to solving IVP’s with $\mathcal{L}$ and more from 10.1 and 10.3
  • Homework assignment:
    – 10.1, p. 565: 1,3,8,9,23,27,29
    – 10.2, p. 577: 2,4,8,9
    – 10.3, p. 583: 13,14,15,19

Week 10 (Mar 26–30) [hmwk due Apr 3].

• Sections covered:
  – Mon.
    10.2: solving systems with $\mathcal{L}$; Theorem 2 (dividing by $s$)
    10.3: Theorem 1 ($s$-axis translation), IVPs
    10.5: intro to unit step function, piecewise functions, and graphing
  – Wed:
    10.3: more IVPs;
    10.4: Theorem 2 (differentiation of Laplace transforms);
    10.5: Theorem 1 ($t$-axis translation), $\mathcal{L}$ (piecewise) intro
  – Fri: 10.5: continue $\mathcal{L}$ (piecewise); IVPs
  • Homework assignment:
    – 10.2, p. 576: 12,14,19,21
    – 10.3, p. 584: 1,3,6,7,28,29,30
    – 10.4, p. 593: 16,17
    – 10.5, p. 601: 1,2,7,10,12,14,19,20,21,31,32 [omit graphs of 31,32]
Week 11 (Apr 2–6) [hmwk due Apr 10]

• Sections covered:
  – Mon: 10.4: convolution; 8.2: undetermined coefficients for systems
  – Wed: 8.2: tank problems, variation of parameters
  – Fri: sketching quadric curves; begin discussion of “gallery” of 7.4

  Note added April 6, 2018: On Friday we had some review of Laplace transforms connected with assigned homework this week instead of introducing the "gallery" and we discussed a "set-up only" tank problem. The gallery" we start next week.

• Homework assignment:
  – Home-made 1, Week 11: Solve the following IVP with $\mathcal{L}$:
    \[ y'' + 7y' + 12y = 6u(t - 5), \quad y(0) = 2, \quad y'(0) = -3 \]
  – Home-made 2, Week 11: Find the inverse Laplace transform of
    \[ F(s) = \frac{e^{-s}}{s} + \frac{2e^{-2s}}{s^2} + \frac{5e^{-5s}}{s} - \frac{4e^{-5s}}{s^2} \]
    and sketch its graph.
  – (Hmmd 1 and 2 are just reinforcement for last week’s work.)
  – 8.2, p. 489: 1, 2, 4, 8, 11, 15(a,b), 17, 21, 23
  – 10.4, p. 593: 1, 2, 5, 6, 36, 37 (For 1, 2, 5, 6 find the convolution two ways: firstly by the definition and secondly by Theorem 1.)
  – Home-made 3: On the same axes sketch the graphs of
    \[ -3x^2 - 3xy + 2y^2 = \pm 1 \]
    using the methods discussed in class and following examples posted on the web page. The numbers are ugly so express all work in terms of numerical approximations accurate to four decimal places. Live it up with your calculator to find eigenvalues and eigenvectors. The emphasis is on making a clear picture with all important points in the picture labeled.
  – Home-made 4: Sketch the graph of
    \[ 3x^2 - 4xy + 5y^2 = 1. \]
    Same directions as above.

Week 12 (Apr 9–13) [hmwk due Apr 17]

• Sections covered:
  – Mon: 1.3: slope fields including examples of logistic type; 2.4: Euler method; a bit more on quadric sketching
  – Wed.: 1.4 and 2.2: solution of logistic-like equations with limiting behavior linked to slope fields; begin 7.4: gallery and method of graphing by “p-and-q-method” in case of complex eigenvalues
  – Fri.: continue 7.4 gallery and method of graphing by “p-and-q-method” in case of real eigenvalues; and a quick new easy-to-learn topic: matrix Euclidean algorithm

• Homework assignment:
  – Contents of worksheets used this Tuesday in recitation are testable but the worksheets do not have to be handed in.
Home-made 1: Tank A is initially filled with 117 gallons of water and contains 13 pounds of salt. Tank B is initially filled with 239 gallons of water and contains 25 pounds of salt. Brine containing 5 lbs/gal of salt is pumped into tank A at a rate of 6 gal/min. Brine containing 7 lbs/gal of salt is pumped into tank B at a rate of 4 gal/min. Well-mixed brine is pumped from tank A to tank B at a rate of 4 gal/min and is allowed to drain from tank A at a rate of 3 gal/min. Well-mixed brine is pumped from tank B to tank A at a rate of 1 gal/min and is allowed to drain from tank B at a rate of 7 gal/min. Set up but do not solve the initial value problem for the amount \( x \) of salt in tank A and amount \( y \) of salt in tank B.

Home-made 2: Solve the initial value problem

\[ y' = (y - 3)(y - 7), \quad y(0) = 4 \]

using separation of variables and evaluate \( \lim_{t \to +\infty} y(t) \). Also answer the question with initial value \( y(0) = 8 \). But now it doesn’t make sense to ask for the limit as \( t \to +\infty \) because the solution ceases to be defined after a certain time. When? Sketch the slope field for the differential equation. Sketch the solutions of the initial value problems on the same axes. Also draw the equilibrium solutions on the same axes and for each indicate whether stable or unstable. (See section 2.2 for vocabulary items “stable”, “unstable” and “equilibrium solution”.)

Home-made 3: Similarly work up the initial value problems

\[ y' = 2y - y^2, \quad y(0) = 0, 3. \]

Home-made 4: Write the solution of the initial value problem

\[
\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}
\]

in the form

\[ e^{\gamma t}(\cos(\omega t + \alpha)p + \sin(\omega t + \alpha)q) \]

where \( 0 \leq \alpha < 2\pi, \omega > 0, \gamma \) is a real number, \( \gamma \pm \omega i \) are the eigenvalues of the coefficient matrix, and \( p \) and \( q \) are some perpendicular vectors satisfying

\[
\begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix} \begin{bmatrix} \gamma & -\omega \\ \omega & \gamma \end{bmatrix}.
\]

Does the trajectory spiral in or out? Does the trajectory rotate counterclockwise or clockwise about the origin? Sketch the trajectory indicating the sense of spiraling and on the same axes graph and label the vectors \( p \) and \( q \).

Home-made 5: Write the solution of the initial value problem

\[
\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}
\]

in the form

\[ e^{\gamma t}(\cosh(\omega t + \alpha)p + \sinh(\omega t + \alpha)q) \]

where \( \omega > 0, \alpha \) and \( \gamma \) are real numbers, \( \gamma \pm \omega \) are the eigenvalues of the coefficient matrix, and \( p \) and \( q \) are some perpendicular vectors.
satisfying
\[
\begin{bmatrix}
1 & 2 \\
5 & 3 \\
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
\end{bmatrix} =
\begin{bmatrix}
p \\
q \\
\end{bmatrix}
\begin{bmatrix}
\gamma & \omega \\
\omega & \gamma \\
\end{bmatrix}.
\]

Sketch the trajectory indicating direction of travel. Plot \(p\) and \(q\) in the picture and also the asymptotes which are parallel to \(p + q\) and \(p - q\) (and by no coincidence are eigenvectors for the coefficient matrix). The trajectory does not cross the asymptotes.

– Home-made 6: Find integers \(x\) and \(y\) such that \(471x + 272y = 1\) using the matrix Euclidean algorithm. Show your row operations.

– 2.2, p. 92: 5,6,9,10 Just find the equilibrium solutions, make a rough sketch of the slope field, and decide whether the equilibrium solutions are stable or unstable on the basis of the slope field sketch. Don’t do more than this. It should not take long to perform these few steps entirely by hand.

– 2.4, p. 114: 4
Week 13 (Apr 16-20) [hmwk due Apr 24]

- **Midterm III, Friday, April 20, 2018**

- Sections covered (actually none from textbook):
  - Mon: Chinese Remainder Theorem; discrete IVPs
  - Wed: follow-up on Monday’s topics as needed; review for MT3

- Homework assignment
  **(LAST OF THE SEMESTER TO BE COLLECTED):**
  - Hmmd 1: Find the smallest positive integer leaving a remainder of 7 upon division by 411 and a remainder of 33 upon division by 325.
  - Hmmd 2: Solve the discrete initial value problem
    \[
    y_{n+2} + 4y_{n+1} + 3y_n = 0, \quad y_0 = 1, \quad y_1 = 0.
    \]
    Lightning review of the Fibonacci rabbit model
    \[
    \begin{array}{c|c|c|c|c|c|c|c}
    n & 1 & 2 & 3 & 4 & 5 & 6 & \cdots \\
    b_n & 1 & 0 & 1 & 1 & 2 & 3 & \cdots \\
    a_n & 0 & 1 & 1 & 2 & 3 & 5 & \cdots \\
    y_n & 1 & 1 & 2 & 3 & 5 & 8 & \cdots \\
    \end{array}
    \]
    \[
    \begin{bmatrix}
    0 & 1 \\
    1 & 1
    \end{bmatrix}
    \begin{bmatrix}
    b_n \\
    a_n
    \end{bmatrix}
    =
    \begin{bmatrix}
    b_{n+1} \\
    a_{n+1}
    \end{bmatrix},
    \begin{vmatrix}
    0 - \lambda & 1 \\
    1 & 1 - \lambda
    \end{vmatrix}
    = \lambda^2 - \lambda - 1
    \]
    \[
    y_{n+2} - y_{n+1} - y_n = 0, \quad y_1 = 1, \quad y_2 = 1.
    \]
    \[
    y_n = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2\sqrt{5}}, \quad \lim_{n \to \infty} \frac{y_{n+1}}{y_n} = \frac{1 + \sqrt{5}}{2} = 1.618033988
    \]
  - Hmmd 3: Analyze the following variant of the Fibonacci rabbit model:
    \[
    \begin{array}{c|c|c|c|c|c|c|c|c}
    n & 1 & 2 & 3 & 4 & 5 & 6 & \cdots \\
    b_n & 1 & 0 & 2 & 2 & 6 & 10 & \cdots \\
    a_n & 0 & 1 & 1 & 3 & 5 & 11 & \cdots \\
    y_n & 1 & 1 & 3 & 5 & 11 & 21 & \cdots \\
    \end{array}
    \]
    In this model adult rabbit pairs add two baby rabbit pairs to the next generation rather than one. Find a second order homogeneous difference equation and initial conditions to characterize \(y_n\), find a closed form formula for \(y_n\), and find \(\lim_{n \to \infty} \frac{y_{n+1}}{y_n}\).
  - Hmmd 4: Analyze the following variant of the Fibonacci rabbit model:
    \[
    \begin{array}{c|c|c|c|c|c|c|c|c|c}
    n & 1 & 2 & 3 & 4 & 5 & 6 & \cdots \\
    b_n & 1 & 0 & 1/2 & 1/2 & 3/4 & 1 & \cdots \\
    a_n & 0 & 1/2 & 1/2 & 3/4 & 1 & 11/8 & \cdots \\
    y_n & 1 & 1/2 & 1 & 5/4 & 7/4 & 19/8 & \cdots \\
    \end{array}
    \]
    In this model only half of the baby rabbit pairs alive at time \(n\) make it to adulthood at time \(n+1\). Find a second order homogeneous difference equation and initial conditions to characterize \(y_n\), find a closed form formula for \(y_n\), and find \(\lim_{n \to \infty} \frac{y_{n+1}}{y_n}\).
Week 14 (Apr 23–27).
  • Sections covered:
    – Mon:
    – Wed:
    – Fri:

Week 15 (Apr 30–May 4) REVIEW.
  • Mon: review
  • Wed: review
  • Fri: review
  • LAST DAY OF INSTRUCTION: Friday, May 4, 2018

Final exam:
Monday, May 7, 2018, noon–3pm