A discrete initial value problem

Solve the following second order linear homogeneous difference equation, taking care to satisfy the indicated initial conditions:

\[ y_{n+2} + 7y_{n+1} + 12y_n = -40, \quad y_0 = 2, \quad y_1 = 1. \]

Brine tank

A very large tank contains 50 lbs of salt dissolved in 400 gallons of water. Brine that contains 4/5 lbs of salt per gallon of water starts entering the tank at time \( t = 0 \) at the rate of 5 gal/min. The mixture leaves the tank at the lower rate of 3 gal/min. Find an expression for the amount of salt in the tank at time \( t \).

Newton’s Law of Heating and Cooling

A steel ball is heated to a temperature of 200°C and at time \( t = 0 \) is placed in water maintained at 20°C. At \( t = 5 \) minutes the temperature of the ball is 110°C. (i) Find the temperature \( y \) of the ball at time \( t \). Start from the differential equation. (ii) Also find the time at which the temperature of the steel ball equals 50°C, reporting your answer to 4 decimal places of accuracy.

Convolution

Find the convolution of \( e^t \) and

\[ f(t) = \begin{cases} 0 & \text{for } 0 \leq t < 1, \\ 1 & \text{for } 1 \leq t < 2, \\ 0 & \text{for } t \geq 2 \end{cases} \]

using Laplace transforms.

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