Questions

1. Given \( f(x) = \frac{2}{x-3} \) and \( g(x) = \frac{1}{x} \)
   a. Find \( (f \circ g)(x) \).  
   b. Find the domain of \( (f \circ g)(x) \).

2. Given \( f(x) = \frac{3x+1}{2x-1} \)
   a. Find inverse.  
   b. Find domain.  
   c. Find range.

3. Solve \( 4^{x^2} = \frac{1}{8^{2x}} \)

4. If \( 3^x = 4 \) what does \( 81^{x-1} \) equal?

5. Solve: \( \frac{4e^{x+4}}{5} = 1 \)

6. Write as a single log: \( 3 + 2 \cdot \ln(x-1) - \log(x^2-1) \)

7. Solve: \( \frac{1}{2} \cdot \log_4(x+18) - 1 = \log_{16}(x+3) \)

8. Given the functions \( f(x) = \frac{5}{x} \) and \( g(x) = \frac{x}{x+1} \):
   a. Find the composition \( (f \circ g)(x) \) and simplify.
   b. What is the domain of \( (f \circ g)(x) \)?

9. Given the function \( f(x) = 3 - \frac{x-1}{x} \):
   a. Find the domain of \( f(x) \).
   b. Find the inverse \( f^{-1}(x) \)
   c. Find the range of \( f(x) \).

10. Solve: \( 2^{2x-3} = \frac{1}{3^{x-1}} \).

11. Solve: \( \log_x 5 = -2 \)

12. Solve: \( 3 - \log_2(x-4) + \log_4(x+4) = 0 \)

13. Find the amount of money you would need to invest in order to have a total amount of $10,000 after 5 years compounded continuously at a rate of 4% per year.

14. Find the effective rate of interest on an account whose annual rate is 12% compounded quarterly.
Solutions

1. Given \( f(x) = \frac{2}{x-3} \) and \( g(x) = \frac{1}{x} \)

   a. Find \( (f \circ g)(x) \).

      \[
      (f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{2}{\left(\frac{1}{x}\right)-3} = \frac{2}{\frac{1}{x} - 3} = \frac{2}{\frac{1}{x} - \frac{3}{1}} = \frac{2}{\frac{1-3x}{x}} = \frac{2x}{1-3x}
      \]

      Ans: \( (f \circ g)(x) = \frac{2x}{1-3x} \)

   b. Find the domain of \( (f \circ g)(x) \).

      Domain of \( g(x) \): \( x \neq 0 \)

      \[
      \frac{1-3x}{x} = 0 \quad \Rightarrow \quad 1-3x = 0 \quad \Rightarrow \quad x = \frac{1}{3}
      \]

      Domain of \( \frac{2x}{1-3x} \): \( x = \frac{1}{3} \)

      So, domain of \( (f \circ g)(x) \) is \( \left\{ x \mid x \neq \frac{1}{3}, 0 \right\} \)

2. Given \( f(x) = \frac{3x+1}{2x-1} \)

   a. Find the inverse of \( f(x) \).

      Exchange the \( x \) and \( y \) and solve for \( y \):

      \[
      f(x): \quad y = \frac{3x+1}{2x-1} \\
      f^{-1}(x): \quad x = \frac{3y+1}{2y-1}
      \]

      \[
      x(2y-1) = 3y + 1 \\
      2xy - x = 3y + 1 \\
      2xy - 3y = x + 1 \\
      y(2x-3) = x + 1 \\
      y = \frac{x+1}{2x-3} = f^{-1}(x)
      \]

      Ans: \( f^{-1}(x) = \frac{x+1}{2x-3} \)
b. Find the domain of \( f(x) = \frac{3x+1}{2x-1}. \) 

\[
2x - 1 = 0 \\
x = \frac{1}{2}
\]

Ans: \( \{ x | x \neq \frac{1}{2} \} \)

c. Find the range of \( f(x) \).

Ans: \( \{ y | y \neq \frac{3}{2} \} \)

Range of \( f(x) \) is the domain of \( f(x)^{-1} \). Domain of \( f(x)^{-1} = \frac{x+1}{2x-3} \) is \( \{ x | x \neq \frac{3}{2} \} \), so range of \( f(x) \) is \( \{ y | y \neq \frac{3}{2} \} \).

3. Solve \( 4^{x^2} = \frac{1}{8^{2x}} \) 

Ans: \( x = 0 \) or \( x = -3 \)

If possible, make all the bases the same and then equate the exponents.

\[
4^{x^2} = \frac{1}{8^{2x}} \\
\left(2^2\right)^{x^2} = \frac{1}{\left(2^3\right)^{2x}} \\
2^{2x^2} = \frac{1}{2^{6x}} \\
2^{2x^2} = 2^{-6x} \\
2x^2 = -6x \\
x^2 + 3x = 0 \\
x(x + 3) = 0 \\
x = 0 \text{ or } x = -3
4. If $3^x = 4$ what does $81^{x-1}$ equal?

Here are two ways to solve this:

\[
\begin{align*}
3^x &= 4 \\
\log_3(3^x) &= \log_3(4) \\
x\log_3(3) &= \log_3(4) \\
x &= \log_3(4) \\
\text{replace } x \text{ with } \log_3(4) \text{ in } 81^{x-1}
\end{align*}
\]

\[
81^{x-1} ightarrow 81^{\log_3(4)-1}
\]
\[
= 81^{\log_3(4)} \cdot 81^{-1}
\]
\[
= \left(\frac{3^4}{81}\right)^{\log_3(4)}
\]
\[
= \left(3^4\right)^{\log_3(4)}
\]
\[
= 81^{\log_3(4^4)}
\]
\[
= 81^{4\log_3(4)}
\]
\[
= 81^4
\]
\[
= \frac{81^4}{81}
\]
\[
= \frac{256}{81}
\]

**Ans:** $81^{x-1} = \frac{256}{81}$

5. Solve: \[\frac{4e^{x+4}}{5} = 1\]

\[
\begin{align*}
4e^{x+4} &= 5 \\
e^{x+4} &= \frac{5}{4} \\
x + 4 &= \ln\left(\frac{5}{4}\right) \\
x &= \ln\left(\frac{5}{4}\right) - 4 \\
&\approx -3.8
\end{align*}
\]

**Ans:** -3.8
6. Write as a single log: \[3 + 2 \cdot \ln(x - 1) - \log(x^2 - 1)\]

\[
3 + 2 \cdot \ln(x - 1) - \log(x^2 - 1) = \ln(e^3) + 2 \cdot \ln(x - 1) - \frac{\ln(x^2 - 1)}{\ln(10)}
\]

\[
= \ln(e^3) + 2 \cdot \ln(x - 1) - \frac{1}{\ln(10)} \cdot \ln(x^2 - 1)
\]

\[
\approx \ln(e^3) + 2 \cdot \ln(x - 1) - 0.4 \cdot \ln(x^2 - 1)
\]

\[
\approx \ln(e^3) + \ln(x - 1)^2 - \ln(x^2 - 1)^{0.4}
\]

\[
\approx \ln \left( \frac{e^3 (x - 1)^2}{(x^2 - 1)^{0.4}} \right)
\]

**Ans:** \[\ln \left( \frac{e^3 (x - 1)^2}{(x^2 - 1)^{0.4}} \right)\]

7. Solve \[\frac{1}{2} \cdot \log_4 (x + 18) - 1 = \log_{16} (x + 3)\]

\[
\frac{1}{2} \cdot \log_4 (x + 18) - 1 = \log_{16} (x + 3)
\]

Get the logs together and then change to an exponential:

\[
\frac{1}{2} \cdot \log_4 (x + 18) = \log_{16} (x + 3) + 1
\]

\[
= \frac{\log_4 (x + 18)}{\log_4 (16)} = 1
\]

\[
\frac{1}{2} \cdot \log_4 (x + 18) = \frac{\log_4 (x + 3)}{\log_4 (4^2)} = 1
\]

\[
\frac{1}{2} \cdot \log_4 (x + 18) = \frac{\log_4 (x + 3)}{2} = 1
\]

\[
\log_4 (x + 18) - \log_4 (x + 3) = 2
\]

\[
\log_4 \left( \frac{x + 18}{x + 3} \right) = 2
\]

\[
\frac{x + 18}{x + 3} = 16
\]

\[
x + 18 = 16x + 48
\]

\[-30 = 15x
\]

\[-2 = x
\]

**Ans:** \[x = -2\]
8. Given the functions $f(x) = \frac{5}{x}$ and $g(x) = \frac{x}{x+1}$:

a. Find the composition $(f \circ g)(x)$ and simplify.  
   \[ (f \circ g)(x) = f(g(x)) = f\left(\frac{x}{x+1}\right) = \frac{5}{\frac{x}{x+1}} = \frac{5x+5}{x} \]

b. What is the domain of $(f \circ g)(x)$?  
   \[ \text{Ans: Domain of } \{x | x \neq 1 \text{ and } x \neq 0\} \]

9. Given the function $f(x) = 3 - \frac{x-1}{x}$:

a. Find the domain of $f(x)$.  
   \[ \text{Ans: Domain of } f(x) : \{x | x \neq 0\} \]

   The denominator of $f(x) = \frac{1-x}{x} + 2$ cannot be 0. So, $x \neq 0$.

b. Find the inverse $f(x)^{-1}$  
   \[ \text{Ans: } f(x)^{-1} = \frac{1}{x-2} \]

   Exchange $x$ and $y$ and then simplify:

   \[
   f(x) = 3 - \frac{x-1}{x} \\
   y = 3 - \frac{x-1}{x} \\
   x = 3 - \frac{y-1}{y} \\
   x = 3 - \frac{y-1}{y} \\
   xy = 3y - (y-1) \\
   xy = 3y - y + 1 \\
   xy = 2y + 1 \\
   xy - 2y = 1 \\
   y(x-2) = 1 \\
   y = \frac{1}{x-2} \\
   \]
c. Find the range of \( f(x) \).

\[ \text{Ans: Range of } f(x) : \{ y \mid y \neq 2 \} \]

The range of \( f(x) \) is the domain of \( f(x)^{-1} \).

The denominator of \( f^{-1}(x) = \frac{1}{x-2} \) cannot be 0. So, \( x \neq 2 \)

10. Solve: \( 2^{2x-3} = \frac{1}{3^{x-1}} \).

\[ \text{Ans: } x = \frac{3 \ln 2 + \ln 3}{2 \ln 2 + \ln 3} \approx 1.3 \]

Take the log of both sides and then use the product property to get the \( x \) out of the exponent.

\[
\ln \left( 2^{2x-3} \right) = \ln \left( \frac{1}{3^{x-1}} \right) \\
(2x - 3) \ln 2 = -(x - 1) \ln 3 \\
(2 \ln 2) x - 3 \ln 2 = (-2 \ln 3) x + \ln 3 \\
(2 \ln 2) x + (\ln 3) x = 3 \ln 2 + \ln 3 \\
[ (2 \ln 2 + \ln 3) ] x = 3 \ln 2 + \ln 3 \\
x = \frac{3 \ln 2 + \ln 3}{2 \ln 2 + \ln 3} \approx \frac{3.17805383}{2.48490665} \approx 1.278942946 \approx 1.3
\]

11. Solve: \( \log_x 5 = -2 \)

\[ \text{Ans: } x = \frac{\sqrt{5}}{5} \approx 0.447213595 \approx 0.4 \]

Convert to an exponential and use logs to solve:

\[
\log_x 5 = -2 \\
x^{-2} = 5 \\
\frac{1}{x^2} = 5 \\
\frac{1}{5} = x^2 \\
\sqrt{\frac{1}{5}} = \sqrt{x^2} \\
x = \pm \sqrt{\frac{1}{5}} = \pm \sqrt{\frac{1}{5} \cdot \frac{\sqrt{5}}{\sqrt{5}}} = \pm \frac{\sqrt{5}}{5}
\]

Since the base of a log must be positive, we discard the negative solution.
12. Solve: \(3 - \log_2(x - 4) + \log_4(x - 4) = 0\).  
\[\text{Ans: } x = 68\]

First, use the change of base formula to convert the logs to the same base.

\[
\begin{align*}
3 - \log_2(x - 4) + \log_4(x - 4) &= 0 \\
3 - \log_2(x - 4) + \frac{\log_2(x - 4)}{\log_2 4} &= 0 \\
3 - \log_2(x - 4) + \frac{1}{2} \log_2(x - 4) &= 0 \\
6 &= \log_2(x - 4) \\
2^6 &= x - 4 \\
68 &= x
\end{align*}
\]

13. Find the amount of money you would need to invest in order to have a total amount of $10,000 after 5 years compounded continuously at a rate of 4\% per year.  
\[\text{Ans: } $8,187\]

\[
\begin{align*}
A &= Pe^{rt} \\
10,000 &= Pe^{0.04 \cdot 5} \\
10,000 &= Pe^{0.2} \\
\frac{10,000}{1.221402758} &= P \\
8,187.307531 &= P
\end{align*}
\]
14. Find the effective rate of interest on an account whose annual rate is 12% compounded quarterly.

**Ans:** Effective rate $= 12.6\%$

**Effective Rate of Interest** is the equivalent annual simple rate of interest that would yield the same amount as compounding after 1 year.

**Simple Interest:** $I = Pr_{\text{simple}}t$

**Compounding:** $A = P\left(1 + \frac{r_{\text{compounding}}}{n}\right)^{nt}$ and $I = A - P$

The time is one year so we replace $t = 1$.

The simple interest rate is given as 12% so we replace $r_{\text{compounding}} = 0.12$.

The compounding is quarterly so $n = 4$.

**Simple Interest = Compound Interest**

$$P r_{\text{simple}}t = P\left(1 + \frac{r_{\text{compounding}}}{n}\right)^{nt} - P$$

$$P \cdot r_{\text{simple}} \cdot 1 = P\left(1 + \frac{0.12}{4}\right)^{4\cdot1} - P$$

$$P \cdot r_{\text{simple}} \cdot 1 = \frac{P(1 + 0.03)^4}{P} - \frac{P}{P}$$

$$r_{\text{simple}} \approx (1.03)^4 - 1$$

$$r_{\text{simple}} \approx 1.12550881 - 1$$

$$r_{\text{simple}} \approx 0.12550881$$

So, the effective rate of interest is about 12.550881%, which rounds to 12.6%.

We can check this by calculating the 12.550881% simple interest on, say, $100 and the 12% compounded monthly interest on $100.

$$I = P r_{\text{simple}}t$$

$$I \approx 100 \cdot 0.12550881 \cdot 1$$

$$I \approx 12.55$$

$$I = A - P$$

$$I = P\left(1 + \frac{r_{\text{compounding}}}{n}\right)^{nt} - P$$

$$I = 100\left(1 + \frac{0.12}{4}\right)^{4\cdot1} - 100$$

$$= 100(1.03)^4 - 100$$

$$\approx 100 \cdot 1.12550881 - 100$$

$$\approx 112.550881 - 100$$

$$\approx 12.55$$