This exam will cover some of the high school algebra that you must know in order to be successful in precalculus. The questions will be like the ones below (most of these were taken from my old exams). The exam probably will have about 10 questions so you will have about 5 minutes to answer each question. The solutions start on page 3.

**Questions**

1. Simplify: $8 - \frac{3^2 \cdot \sqrt{25 - 9} + 12}{8 - 24 \div 6 \cdot 2 - 3 \cdot 2^2}$

2. The triangles shown below are similar. Find the missing length $x$ (to the nearest tenth of a foot), and the missing angles $A$, $B$, $C$, and $D$.

3. Calculate the area of the following shape. It is an equilateral triangle on top of a rectangle. The height of the rectangle is 4 feet. The perimeter of the entire shape is 14 feet. Round your answer to the nearest tenth.

4. Expand and simplify: $(x - 2)^3$

5. Find the quotient and remainder of $3x^4 - 2x^2 + 8x - 1$ divided by $x + 2$.

6. Factor completely: $10x^2 + 3x - 18$

7. Reduce to lowest terms: $\frac{4x^2 - 9}{2x^3 - 7x^2 - 15x}$

8. Perform the indicated operations and simplify: $\frac{1}{x^2 - 9} - \frac{x - 3}{x^2 + 6x + 9}$

9. Solve: $\frac{x}{6} + 12 = 4x + \frac{1}{10}(x - 3) + \frac{1}{2}$

10. What number should be added to $x^2 - \frac{1}{2}x$ to make it a perfect square (that is, to complete the square)?

11. Simplify completely: $\sqrt[4]{32x^{16}y^9}$

12. Rationalize the denominator and simplify: $\frac{1}{3 - \sqrt{2}}$

13. Write in $a + bi$ form: $\frac{2 - i}{3 + i}$
14. Calculate the area of the shape shown.
   Each side is 2 feet in length and the width of the shape is 2 feet. Round your answer to two decimal places.

15. Find the quotient and the remainder of \(4x^5 - 5x^3 + 3x - 1\) divided by \(2x^2 - 1\)

16. Factor completely: \(6x^2 + 5x - 6\)

17. Factor completely: \(2x^3 + 10x^2 + 12x\)

18. Perform the indicated operations and simplify: \(\frac{x + 3}{x^2 + 2x + 1} - \frac{x}{x^2 + x}\)

19. Solve: \(\frac{3x}{4} + 2 = \frac{1}{2}(x - 8) + 5\)

20. Solve: \(12 - 4x = x^2\)

21. Simplify: \(5 - \frac{6 - 3 \cdot 2^3}{24 + 2 \cdot 3 - 2 \cdot 3^2}\)

22. Factor completely: \(30x^3 + 9x^2 - 54x\)

23. Perform the indicated operations and simplify: \(\frac{1}{x^2 - 9} - \frac{2x - 3}{x^2 + 6x + 9}\)

24. Solve: \(\frac{x}{6} + 12 = -4x + \frac{1}{10}(x - 3) + \frac{25}{6}\)

25. Simplify completely: \(\sqrt[3]{16x^{20}y^9}\)

26. A freight train and a passenger train must share the same track. The freight train leaves a station at 9 am and travels 50 mph. One hour later, the passenger train leaves the station and travels 60 mph in the same direction. To avoid a crash, at what time and where (distance down the track) must the railroad dispatcher move the freight train to a side track?

27. One hose can fill a pool by itself in 3 hours. As smaller hose can fill the pool by itself in 5 hours. If both hoses are used to fill the pool, how long with it take?

28. How many ounces of a 30% acid solution must be added to 10 ounces of a 50% solution to produce a 45% solution?

29. In the triangle, the lengths of the indicated line segments are as follows: \(ac = 3\), \(cd = 4\), \(ae = 2\)

   Find the length of line segment \(ab\).

30. Solve: \(2 \mid 3x - 6 \mid - 7 > 5\)

31. Find the approximate area of this shape. Assume the top is a half circle and all sides of the hexagon are of equal length. Round your answer to 2 decimal places.
Solutions

1. Simplify: \(8 - \frac{3^2 \cdot \sqrt{25-9} + 12}{8 - 24 \div 6 \cdot 2 - 3 \cdot 2^2}\)

Follow the order of operations for simplifying arithmetic expressions:

**Step 1** Simplify expressions inside grouping symbols, which include parentheses ( ), brackets [ ], the fraction bar \(\frac{a}{b}\), absolute value bars \(|\ |\), and radicals \(n\sqrt{a}\).

**Step 2** Simplify exponents, square roots, and absolute values.

**Step 3** Simplify multiplication, square roots, and absolute values.

**Step 4** Simplify addition and subtraction, working left to right.

\[
8 - \frac{3^2 \cdot \sqrt{25-9} + 12}{8 - 24 \div 6 \cdot 2 - 3 \cdot 2^2} = 8 - \frac{3^2 \cdot \sqrt{16} + 12}{8 - 24 \div 6 \cdot 2 - 3 \cdot 2^2}
\]
\[
= 8 - \frac{9 \cdot \sqrt{16} + 12}{8 - 24 \div 6 \cdot 2 - 3 \cdot 2^2}
\]
\[
= 8 - \frac{9 \cdot 4 + 12}{8 - 24 \div 6 \cdot 2 - 3 \cdot 2^2}
\]
\[
= 8 - \frac{36 + 12}{8 - 24 \div 6 \cdot 2 - 3 \cdot 2^2}
\]
\[
= 8 - \frac{48}{8 - 24 \div 6 \cdot 2 - 3 \cdot 2^2}
\]
\[
= 8 - \frac{48}{8 - 24 \div 6 \cdot 2 - 3 \cdot 4}
\]
\[
= 8 - \frac{48}{8 - 8 - 3 \cdot 4}
\]
\[
= 8 - \frac{48}{8 - 8 - 12}
\]
\[
= 8 - \frac{48}{-12}
\]
\[
= 8 + 4
\]
\[
= 12
\]

**Ans:** 12
2. The triangles shown below are similar. Find the missing length \( x \) (to the nearest tenth of a foot), and the missing angles \( A, B, C, \) and \( D \).

First, find \( D \): Since the sum of the angles of any triangle is 180, we have \( 40 + 30 + D = 180 \), so \( D = 110 \). Since the triangles are similar, the corresponding angles are equal. Therefore, \( A = 40 \), \( B = 110 \), \( C = 30 \). Since the triangles are similar, the ratios of corresponding sides are equal. That is,

\[
\frac{15}{24.6} = \frac{x}{28.2}
\]

\( 15x = (24.6)(28.2) \)

\( x = \frac{(24.6)(28.2)}{15} \)

\( x = 46.248 \approx 46.2 \)

**Ans:** \( x = 46.2 \) ft, \( A = 40 \) deg, \( B = 110 \) deg, \( C = 30 \) deg, \( D = 110 \) deg

3. Calculate the area of the following shape. It is an equilateral triangle on top of a rectangle. The height of the rectangle is 4 feet. The perimeter of the entire shape is 14 feet. Round your answer to the nearest tenth.

**Area**

\( \text{Area}_{\text{total}} = \text{Area}_{\text{rectangle}} + \text{Area}_{\text{triangle}} \)

We know the formulas for areas:

\( \text{Area}_{\text{rectangle}} = \text{length} \times \text{width} \)

\( \text{Area}_{\text{triangle}} = \frac{1}{2} \text{base} \times \text{height} \)

To find the area of the rectangle we need to figure out the width. Since the triangle is equilateral, all three of its sides are the same. So, let \( x = \) side of triangle.

We are given the perimeter is 14 feet. We can write:

\( \text{Perimeter} = \text{side of triangle} + \text{height of rectangle} + \text{width of rectangle} + \text{height of rectangle} + \text{side of triangle} \)

The width of the rectangle is the same as the base of the triangle, which is \( x \). So, we can write:

\( 14 = x + 4 + x + 4 + x \)

\( 14 = 3x + 8 \)

\( 6 = 3x \)

\( 2 = x \)
Now we know the width of the rectangle is 2 and the length is 4:

\[ \text{Area}_{\text{rectangle}} = \text{length} \times \text{width} = 4 \times 2 = 8 \]

To find the area of the triangle we need to know its base (it is \(x = 2\)) and its height. To find the height we note that we can form a right triangle where the hypotenuse is \(x\) (length of the side of the triangle), the base is 1 (half the bottom side of the triangle) and the height, which we want to find.

\[
2^2 = 1^2 + h^2 \\
4 = 1 + h^2 \\
3 = h^2 \\
\pm \sqrt{3} = \sqrt{h^2} \\
\pm \sqrt{3} = h
\]

Since \(h\) is a height, we ignore the negative root. So, we can write:

\[
\text{Area}_{\text{triangle}} = \frac{1}{2}bh = \frac{1}{2} \times 2 \times \sqrt{3} = \sqrt{3} \approx 1.7
\]

\[
\text{Area}_{\text{total}} = \text{Area}_{\text{rectangle}} + \text{Area}_{\text{triangle}} = 8 + 1.7 = 9.7
\]

**Ans:** Area = 9.7 sq ft

4. Expand and simplify: \((x - 2)^3\)

You cannot cube each term inside the parentheses. You must expand this and then use FOIL.

\[
(x - 2)^3 = (x - 2)(x - 2)(x - 2) = (x^2 - 2x - 2x + 4)(x - 2)
\]

\[
= (x^2 - 4x + 4)(x - 2)
\]

\[
= x^3 - 4x^2 + 4x - 2x^2 + 8x - 8
\]

\[
= x^3 - 6x^2 + 12x - 8
\]

**Ans:** \(x^3 - 6x^2 + 12x - 8\)
5. Find the quotient and remainder of \(3x^4 - 2x^2 + 8x - 1\) divided by \(x + 2\).

\[
\begin{array}{c|c}
\text{Dividend} & 3x^4 - 2x^2 + 8x - 1 \\
\text{Divisor} & x + 2 \\
\end{array}
\]

\[
\begin{array}{c}
3x^3 - 6x^2 + 10x - 12 \\
\hline \\
\frac{3x^4 + 0x^3 - 2x^2 + 8x - 1}{x + 2} \\
\hline \\
-\left(3x^4 + 6x^3\right) \\
\hline \\
-6x^3 - 2x^2 \\
\hline \\
-\left(-6x^3 - 12x^2\right) \\
\hline \\
10x^2 + 8x \\
\hline \\
-\left(10x^2 + 20x\right) \\
\hline \\
-12x - 1 \\
\hline \\
-\left(-12x - 24\right) \\
\hline \\
23
\end{array}
\]

\text{Ans: Quotient} = 3x^3 - 6x^2 + 10x - 12 \quad \text{Ans: Remainder} = 23

6. Factor completely: \(10x^2 + 3x - 18\)

First, look for the factors that are common to each term. There are none.

Since \(10x^2 + 3x - 18\) has the form \(ax^2 + bx + c\) find two integers whose product is \(a\cdot c\) and whose sum is \(b\), replace the \(bx\) term using these integers, and factor by grouping.

For this problem \(a\cdot c = (10)(-18) = -180\).

Here are the pairs of positive integers factors whose product is 180:
1•180, 2•90, 3•60, 4•45, 5•36, 6•30, 7•n/a, 8•n/a, 9•20, 10•18, 11•n/a, 12•15

The pair whose sum or difference is 3 is 12 and 15. That is, \(-12\cdot15 = -180\) and \(-12 + 15 = 3\)

Now, replace the middle term using the two integers we just found.

\[
10x^2 + 3x - 18 \\
10x^2 - 12x + 15x - 18
\]

Now, factor by grouping:

\[
\left(10x^2 - 12x\right) + \left(15x - 18\right) \\
2x(5x - 6) + 3(5x - 6) \\
(5x - 6)(2x + 3)
\]

\text{Ans:} \ (5x - 6)(2x + 3)
7. Reduce to lowest terms: \( \frac{4x^2 - 9}{2x^3 - 7x^2 - 15x} \)

Factor numerator and denominator and cancel common terms:

\[
\frac{4x^2 - 9}{2x^3 - 7x^2 - 15x} = \frac{(2x - 3)(2x + 3)}{x(2x^2 - 7x - 15)} = \frac{(2x - 3)(2x + 3)}{x(2x + 3)(x - 5)} = \frac{2x - 3}{x(x - 5)}
\]

Ans: \( \frac{2x - 3}{x(x - 5)} \)

8. Perform the indicated operations and simplify: \( \frac{1}{x^2 - 9} - \frac{x - 3}{x^2 + 6x + 9} \)

First, find the LCD. To do that, factor each denominator and then multiply the largest group of each factor:

\[
x^2 - 9 = (x - 3)(x + 3)
\]

\[
x^2 + 6x + 9 = (x + 3)(x + 3)
\]

\[
LCD = (x - 3)(x + 3)(x + 3)
\]

Now convert each fraction into an equivalent fraction but with the LCD as the denominator:

\[
\frac{1}{(x - 3)(x + 3)} \cdot \frac{(x + 3)}{(x + 3)} - \frac{(x - 3)}{(x + 3)(x + 3)} \cdot \frac{(x - 3)}{(x - 3)} = \frac{(x + 3) - (x - 3)(x - 3)}{(x + 3)(x + 3)(x - 3)}
\]

\[
= \frac{(x + 3) - (x^2 - 6x + 9)}{(x + 3)(x + 3)(x - 3)}
\]

\[
= \frac{x + 3 - x^2 + 6x - 9}{(x + 3)(x + 3)(x - 3)}
\]

\[
= \frac{-x^2 + 7x - 6}{(x + 3)(x + 3)(x - 3)}
\]

\[
= -\frac{x^2 - 7x + 6}{(x + 3)(x + 3)(x - 3)}
\]

\[
= -\frac{(x - 6)(x - 1)}{(x + 3)(x + 3)(x - 3)}
\]

Ans: \( -\frac{(x - 6)(x - 1)}{(x + 3)(x + 3)(x - 3)} \)
9. Solve: \( \frac{x}{6} + 12 = 4x + \frac{1}{10}(x - 3) + \frac{1}{2} \)

First, clear the fractions by multiplying each term by the LCD of all the fractions:

6 = 2 \cdot 3, 10 = 2 \cdot 5, and 2 is prime so the LCD is 2 \cdot 3 \cdot 5 = 30.

Then solve using the properties of equality.

\[
30 \left( \frac{x}{6} \right) + 30 \cdot (12) = 30 \cdot (4x) + 30 \cdot \left[ \frac{1}{10} (x - 3) \right] + 30 \cdot \left( \frac{1}{2} \right)
\]

\[
5x + 360 = 120x + 3(x - 3) + 15
\]

\[
5x + 360 = 120x + 3x - 9 + 15
\]

\[
5x + 360 = 123x + 6
\]

\[
354 = 118x
\]

\[
x = 3
\]

Ans: \( x = 3 \)

We can check this answer by replacing \( x \) with 3 in the original equation and seeing if a true statement results.

\[
\frac{x}{6} + 12 = 4x + \frac{1}{10}(x - 3) + \frac{1}{2}
\]

\[
\frac{3}{6} + 12 = 4 \cdot 3 + \frac{1}{10}(3 - 3) + \frac{1}{2}
\]

\[
\frac{1}{2} + 12 = 12 + \frac{1}{2} \quad \text{yes}
\]

10. What number should be added to \( x^2 - \frac{1}{2}x \) to make it a perfect square (that is, to complete the square)?

To complete the square we add the square of half the coefficient of the \( x \)-term. In this case, the coefficient of the \( x \)-term is \( -\frac{1}{2} \). So, we need to add: \( \left[ \frac{1}{2} \left( -\frac{1}{2} \right) \right]^2 = \left[ -\frac{1}{4} \right]^2 = \frac{1}{16} \)

Ans: \( \frac{1}{16} \)
11. Simplify completely: \( 4\sqrt[4]{32x^{16}y^9} \)

Write 32 as \(2^5\).

\[
4\sqrt[4]{32x^{16}y^9} = 4\sqrt[4]{2^5x^{16}y^9}
\]

Since this is a 4th root, divide each exponent by 4. The quotients are the exponents of the factors outside the radical while the remainders are the exponents of the factors inside the radical.

\[
2^5 : 5 \div 4 = 1 \text{ with remainder 1}
\]

\[
x^{16} : 16 \div 4 = 4 \text{ with remainder 0}
\]

\[
y^9 : 9 \div 4 = 2 \text{ with remainder 1}
\]

So, \(4\sqrt[4]{32x^{16}y^9} = 2^1 \cdot x^4 \cdot y^2 \cdot 4\sqrt{2y} = 2x^4y^2 \cdot 4\sqrt{2y} \)

\[\text{Ans: } 2x^4y^2 \cdot 4\sqrt{2y}\]

12. Rationalize the denominator and simplify: \(\frac{1}{3 - \sqrt{2}}\)

Multiply top and bottom by the conjugate of the bottom.

\[
\frac{1}{3 - \sqrt{2}} \cdot \frac{3 + \sqrt{2}}{3 + \sqrt{2}} = \frac{3 + \sqrt{2}}{9 - 3\sqrt{2} + 3\sqrt{2} - \sqrt{4}} = \frac{3 + \sqrt{2}}{9 - 2} = \frac{3 + \sqrt{2}}{7}
\]

\[\text{Ans: } \frac{3 + \sqrt{2}}{7}\]

13. Write in \(a + bi\) form: \(\frac{2 - i}{3 + i}\).

Multiply the numerator and the denominator by the complex conjugate of the denominator.

\[
\frac{2 - i}{3 + i} \cdot \frac{3 - i}{3 - i} = \frac{6 - 2i - 3i + i^2}{9 - 3i + 3i - i^2} = \frac{6 - 5i - 1}{9 - (-1)} = \frac{5 - 5i}{10} = \frac{1 - i}{2} = \frac{1}{2} + \frac{-1}{2}i
\]

\[\text{Ans: } \frac{1}{2} + \frac{-1}{2}i\]
14. Calculate the area of the shape shown. 

Each side is 2 feet in length and the width of the shape is 2 feet. Round your answer to two decimal places.

The shape is made up of 2 triangles and a rectangle so we can write the area as:

\[ \text{Area}_{\text{total}} = \text{Area}_{\text{rectangle}} + \text{Area}_{\text{top triangle}} + \text{Area}_{\text{bottom triangle}} \]

\[ \text{Area}_{\text{rectangle}} = \text{Length} \times \text{Width} \]

Since the shape is 2 feet wide, the width of the rectangle is 2.

Since the length of each side is 2 feet, the length of the rectangle is 2.

\[ \text{Area}_{\text{rectangle}} = 2 \times 2 = 4 \]

\[ \text{Area}_{\text{top triangle}} = \frac{1}{2} \text{Base} \times \text{Height} \]

Since the shape is 2 feet wide, the base of the triangle is 2.

To find the height of the triangle, use Pythagoras:

\[ c^2 = a^2 + b^2 \]

\[ 2^2 = h^2 + h^2 \]

\[ 3 = 2h^2 \]

\[ \pm \sqrt{3} = h \]

Since the height must be positive, we have \( h = \sqrt{3} \).

\[ \text{Area}_{\text{top triangle}} = \frac{1}{2} \times 2 \times \sqrt{3} = \sqrt{3} \approx 1.73 \]

The top and bottom triangles are congruent (the same size) so their areas are the same.

\[ \text{Area}_{\text{total}} = \text{Area}_{\text{rectangle}} + 2 \times \text{Area}_{\text{top triangle}} \]

\[ = 4 + 1.73 + 1.73 \]

\[ = 7.46 \text{ feet} \]

\[ \text{Ans: Area} \approx 7.46 \text{ feet} \]
15. Find the quotient and the remainder of \(4x^5 - 5x^3 + 3x - 1\) divided by \(2x^2 - 1\)

Do this by long division:

\[
\begin{array}{c|ccccc}
& 2x^3 & -1.5x \\
\hline
2x^2 - 1 & 4x^5 & -5x^3 & +3x & -1 \\
& - (4x^5 - 2x^3) \\
& & -3x^3 & +3x & -1 \\
& & -(-3x^3 + 1.5x) \\
& & & & 1.5x - 1
\end{array}
\]

**Ans:** Quotient = \(2x^3 - 1.5x\)  
**Ans:** Remainder = \(1.5x - 1\)

16. Factor completely: \(6x^2 + 5x - 6\)  

**Ans:** \((2x + 3)(3x - 2)\)

First, check for a common factor. There is none, other than 1 or \(-1\).

Second, count the number of terms. There are 3. Check to see if this might be a perfect square trinomial by seeing if the first and last terms are perfect squares. They are not, so this is not a perfect square trinomial.

Third, note that the coefficient of the dominant term (the term with the highest degree) is not 1 or \(-1\) so we will use the ac (factor by grouping) method to factor this.

Step 1: Find two integers whose product is \(6 \cdot (-6) = -36\) and whose sum is +5. Here are the integer factors of 36: \(1 \times 36, 2 \times 18, 3 \times 12, 4 \times 9, 6 \times 6\)

The pair we want is 4 and 9. The sum must be +5 so we will use \(-4\) and 9.

Step 2: Replace the middle term using the numbers from Step 1.  
That is, replace +5x with \(9x - 4x\) (or \(-4x + 9x\)).

\[6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6\]

Step 3: Factor by grouping:

\[6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6 = (6x^2 + 9x) + (-4x - 6) = 3x(2x + 3) + (-2)(2x + 3) = (2x + 3)(3x - 2)\]
17. Factor completely: \(2x^3 + 10x^2 + 12x\)

First, check for a common factor. It is \(2x\) so factor that out of each term.

\[2x^3 + 10x^2 + 12x = 2x(x^2 + 5x + 6)\]

Second, count the number of terms. There are 3. Check to see if this might be a perfect square trinomial by seeing if the first and last terms are perfect squares. They are not, so this is not a perfect square trinomial.

Third, note that the coefficient of the dominant term (the term with the highest degree) is 1 so we will use the Product Sum method to factor this. That is, find two integers whose product is the last term, 6, and whose sum is the coefficient of the middle term, 5. Here are the integer factors of 6: 1 \(\times\) 6 and 2 \(\times\) 3. We will use 2 and 3 since \(2 \times 3 = 6\) and \(2 + 3 = 5\).

\[2x^3 + 10x^2 + 12x = 2x(x^2 + 5x + 6) = 2x(x + 2)(x + 3)\]

18. Perform the indicated operations and simplify: \(\frac{x + 3}{x^2 + 2x + 1} - \frac{x}{x^2 + x}\)

First, we must find the LCD. To do this, factor each denominator and then multiply the largest group of each factor.

\[x^2 + 2x + 1 = (x + 1)(x + 1)\]
\[x^2 + x = x(x + 1)\]

\[\text{LCD} = x(x + 1)(x + 1)\]

Second, convert each fraction so that it has the LCD as its denominator. To do this, multiply the top and bottom of each fraction by the factors the bottom is missing. Then, combine like terms.

\[
\frac{x + 3}{x^2 + 2x + 1} - \frac{x}{x^2 + x} = \frac{x + 3}{(x + 1)(x + 1)} \cdot \frac{x}{x} - \frac{x}{x(x + 1)} \cdot \frac{x + 1}{x + 1}
\]

\[= \frac{x^2 + 3x}{x(x + 1)(x + 1)} - \frac{x^2 + x}{x(x + 1)(x + 1)}
\]

\[= \frac{2x}{x(x + 1)(x + 1)}
\]

\[\text{Ans: } \frac{2}{(x + 1)(x + 1)}\]
19. Solve: \( \frac{3x}{4} + 2 = \frac{1}{2}(x - 8) + 5 \)

This equation has fractions so the first step is to find the LCD and then multiply each side of the equation by the LCD.

The LCD is 4 (the smallest number divisible by both 2 and 4).

\[
4 \cdot \left[ \frac{3x}{4} + 2 \right] = 4 \cdot \left[ \frac{1}{2}(x - 8) + 5 \right] \\
4 \cdot \left[ \frac{3x}{4} \right] + 4 \cdot [2] = 4 \cdot \left[ \frac{1}{2}(x - 8) \right] + 4 \cdot [5] \\
3x + 8 = 2(x - 8) + 20 \\
3x + 8 = 2x - 16 + 20 \\
3x + 8 = 2x + 4 \\
x = -4
\]

Ans: \( x = -4 \)

20. Solve: \( 12 - 4x = x^2 \)

This is a degree 2 equation so we put all terms on one side and 0 on the other. Then, we factor and set each factor equal to 0.

\[
12 - 4x = x^2 \\
0 = x^2 + 4x - 12 \\
0 = (x - 2)(x + 6) \\
0 = x - 2 \quad \text{or} \quad 0 = x + 6 \\
2 = x \quad \text{or} \quad -6 = x
\]

Ans: \( x = 2 \) or \( x = -6 \)
21. Simplify: \[ 5 - \frac{6 - 3 \cdot 2^3}{24 \div 2 \cdot 3 - 2 \cdot 3^2} \]

Follow the order of operations for simplifying arithmetic expressions:

**Step 1** Simplify expressions inside grouping symbols, which include parentheses ( ), brackets [ ], the fraction bar \[ \frac{a}{b} \], absolute value bars \[ | \ ], and radicals \[ \sqrt[n]{a} \].

**Step 2** Simplify exponents, square roots, and absolute values.

**Step 3** Simplify multiplication and division, working left to right.

**Step 4** Simplify addition and subtraction, working left to right.

\[
5 - \frac{6 - 3 \cdot 2^3}{24 \div 2 \cdot 3 - 2 \cdot 3^2} = 5 - \frac{6 - 3 \cdot 8}{24 \div 2 \cdot 3 - 2 \cdot 3^2} \\
= 5 - \frac{6 - 24}{24 \div 2 \cdot 3 - 2 \cdot 3^2} \\
= 5 - \frac{-18}{24 \div 2 \cdot 3 - 2 \cdot 3^2} \\
= 5 - \frac{-18}{24 \div 2 \cdot 3 - 2 \cdot 9} \\
= 5 - \frac{-18}{12 \cdot 3 - 2 \cdot 9} \\
= 5 - \frac{-18}{36 - 18} \\
= 5 - \frac{-18}{18} \\
= 5 + 1 \\
= 6
\]

**Ans:** 6
22. Factor completely: \(30x^3 + 9x^2 - 54x\)

First, factor out the GCF, which is 3x:

\[
30x^3 + 9x^2 - 54x \rightarrow 3x\left(10x^2 + 3x - 18\right)
\]

Since \(10x^2 + 3x - 18\) has the form \(ax^2 + bx + c\), find two integers whose product is \(a\cdot c\) and whose sum is \(b\), replace the \(bx\) term using these integers, and factor by grouping.

For this problem \(a\cdot c = (10)(-18) = -180\).

Here are the pairs of positive integers whose product is 180:

- 1\(\cdot\)180,
- 2\(\cdot\)90,
- 3\(\cdot\)60,
- 4\(\cdot\)45,
- 5\(\cdot\)36,
- 6\(\cdot\)30,
- 7\(\cdot\)n/a,
- 8\(\cdot\)n/a,
- 9\(\cdot\)20,
- 10\(\cdot\)18,
- 11\(\cdot\)n/a,
- 12\(\cdot\)15

The pair whose sum or difference is 3 is 12 and 15.

That is, \(-12\cdot15 = -180\) and \(-12 + 15 = 3\)

Now, replace the middle term using the two integers we just found.

\[
3x\left(10x^2 + 3x - 18\right) = 3x\left(10x^2 - 12x + 15x - 18\right)
\]

Now, factor by grouping:

\[
3x\left[(10x^2 - 12x) + (15x - 18)\right] = 3x\left[2x(5x - 6) + 3(5x - 6)\right] = 3x(5x - 6)(2x + 3)
\]

**Ans:** \(3x(5x - 6)(2x + 3)\)
23. Perform the indicated operations and simplify: \( \frac{1}{x^2 - 9} - \frac{2x - 3}{x^2 + 6x + 9} \)

First, find the LCD. To do that, factor each denominator and then multiply the largest group of each factor:

- \( x^2 - 9 = (x - 3)(x + 3) \)
- \( x^2 + 6x + 9 = (x + 3)(x + 3) \)

\[ \text{LCD} = (x - 3)(x + 3)(x + 3) \]

Now convert each fraction into an equivalent fraction but with the LCD as the denominator:

\[
\frac{1}{x - 3} \cdot \frac{(x + 3)}{(x + 3)(x + 3)} - \frac{(2x - 3)}{x + 3} \cdot \frac{(x - 3)}{(x + 3)(x + 3)} = \frac{1 - (2x - 3)(x - 3)}{(x - 3)(x + 3)(x + 3)}
\]

\[
= \frac{(x - 3) - (2x^2 - 9x + 9)}{(x - 3)(x + 3)(x + 3)}
\]

\[
= \frac{x + 3 - 2x^2 + 9x - 9}{(x + 3)(x + 3)(x - 3)}
\]

\[
= \frac{-2x^2 + 10x - 6}{(x + 3)(x + 3)(x - 3)}
\]

Now, reduce if possible. The numerator cannot be factored because there are no two integers whose product is \((-2)(-6) = 12\) and whose sum is 10. So, this cannot be reduced.

**Ans:** \( \frac{-2x^2 + 10x - 6}{(x + 3)(x + 3)(x - 3)} \)

24. Solve: \( \frac{x}{6} + 12 = -4x + \frac{1}{10}(x - 3) + \frac{25}{6} \)

First, clear the fractions by multiplying each term by the LCD of all the fractions: \( 6 = 2 \cdot 3 \) and \( 10 = 2 \cdot 5 \) so the LCD is \( 2 \cdot 3 \cdot 5 = 30 \).

Then solve using the properties of equality.

\[
30 \cdot \left( \frac{x}{6} \right) + 30 \cdot (12) = 30 \cdot (-4x) + 30 \cdot \left( \frac{1}{10} (x - 3) \right) + 30 \cdot \left( \frac{25}{6} \right)
\]

\[
5x + 360 = -120x + 3(x - 3) + 125
\]

\[
5x + 360 = -120x + 3x - 9 + 125
\]

\[
5x + 360 = -117x + 116
\]

\[
244 = -122x
\]

\[
-2 = x
\]

We can check this answer by replacing \( x \) with -2 in the original equation and seeing if a true statement results.

**Ans:** \( x = -2 \)
25. Simplify completely: $\sqrt[3]{16x^{20}y^9}$

A quick way to do this is to divide each exponent in the radicand by the index of the radical, 3. The quotient is the exponent outside the radical and the remainder is the exponent inside.

For $2^4$ we have: $\frac{4}{3}$ = $\frac{1}{3}$, For $x^{20}$ we have: $\frac{20}{3}$ = $\frac{6}{3}$ = 2, For $y^{9}$ we have: $\frac{9}{3}$ = 3

$1 \overline{3} 4$
$3 \overline{2} 0$
$3 \overline{1} 8$
$3 \overline{0} 2$
$3 \overline{0} 0$

This gives $2^1x^6y^3 \cdot 3^{2}x^2 = 2x^6y^3 \cdot 3\sqrt{2x^2}$

Ans: $2x^6y^3 \cdot 3\sqrt{2x^2}$

26. A freight train and a passenger train must share the same track. The freight train leaves a station at 9 am and travels 50 mph. One hour later, the passenger train leaves the station and travels 60 mph in the same direction. To avoid a crash, at what time and where (distance down the track) must the railroad dispatcher move the freight train to a side track?

Let $x =$ time of travel for the fast train

$D =$ RT for each train.

$D_{\text{fast}} = R_{\text{fast}} * T_{\text{fast}}$

$D_{\text{fast}} = 60x$

$D_{\text{slow}} = R_{\text{slow}} * T_{\text{slow}}$

$D_{\text{slow}} = 50(x + 1)$

Since the trains leave from the same place and take the same route they travel the same distance.

$D_{\text{fast}} = D_{\text{slow}}$

$60x = 50(x + 1)$

$60x = 50x + 50$

$10x = 50$

$x = 5$

The fast train will catch the slow train in 5 hours. The slow train leaves at 9 am. The fast train leaves 1 hour later, which is 10 am. So, we have 10:00 am + 5:00 hours = 15:00 hours = 3:00 pm.

To find where they will meet, use $D_{\text{fast}} = 60(5) = 300$ miles from the station.

Ans: Time = 3 pm Ans: Distance = 300 miles

27. One hose can fill a pool by itself in 3 hours. As smaller hose can fill the pool by itself in 5 hours. If both hoses are used to fill the pool, how long with it take?

Let $t =$ time (hours) it will take them working together.

Rate of big hose is $\frac{1 \text{ job}}{3 \text{ hour}}$ and it works $t$ hours. So, the part it does in $t$ hours is $\left(\frac{1 \text{ job}}{3 \text{ hour}}\right) \cdot t \text{ hours}$
Rate of small hose is \( \frac{1 \text{ job}}{5 \text{ hour}} \) and it works \( t \) hours. So, the part it does in \( t \) hours is \( \left( \frac{1 \text{ job}}{5 \text{ hour}} \right) \cdot t \text{ hours} \)

In \( t \) hours 1 whole job gets done. So, we can write:

\[
\text{Part done by big hose} + \text{Part done by small hose} = \text{Part done together}
\]

\[
\left( \frac{1 \text{ job}}{3 \text{ hour}} \right) \cdot t \text{ hours} + \left( \frac{1 \text{ job}}{5 \text{ hour}} \right) \cdot t \text{ hours} = \frac{1 \text{ job}}{t \text{ hours}} \cdot t \text{ hours}
\]

\[
\frac{1}{3} t + \frac{1}{5} t = 1
\]

\[
5t + 3t = 15
\]

\[
t = \frac{15}{8}
\]

So, working together the two hoses will take 1 and \( \frac{7}{8} \) hours. \hspace{1cm} \text{Ans:} \frac{7}{8} \text{ hours.}

28. How many ounces of a 30% acid solution must be added to 10 ounces of a 50% solution to produce a 45% solution?

Let \( x \) = amount of 30% solution.

\[
\text{Acid in 30\% soln} + \text{Acid in 50\% soln} = \text{Acid in 45\% soln}
\]

\[
0.30x + 0.50 \cdot 10 = 0.45(x + 10)
\]

\[
0.30x + 5 = 0.45x + 4.5
\]

\[
-0.15x = -0.5
\]

\[
x \approx 3.33
\]

29. Triangles \( \text{acd} \) and \( \text{abe} \) are similar so the ratios of the corresponding sides are equal.

That is, \( \frac{ac}{ab} = \frac{cd}{be} = \frac{da}{ea} \). Given the dimensions, we can write \( \frac{ab}{3} = \frac{2}{ad} \). To find the length of \( ad \) we use the Pythagorean Theorem: \( \overline{ad}^2 = \overline{ac}^2 + \overline{cd}^2 = 3^2 + 4^2 = 9 + 16 = 25 \). So, \( \overline{ad} = \sqrt{25} = 5 \).

Now we can write \( \frac{ab}{3} = \frac{2}{5} \). Solving, we get \( \frac{ab}{6} = 1.2 \). \hspace{1cm} \text{Ans:} 1.2.

30. Get the absolute value by itself first.

Then use the definition to write two equations:

\[
2 \left| 3x - 6 \right| - 7 > 5
\]

\[
2 \left| 3x - 6 \right| > 12
\]

\[
3x - 6 > 6
\]

\[
3x - 6 < -6 \quad \text{or} \quad 3x - 6 > 6
\]

\[
3x < 0 \quad \text{or} \quad 3x > 12
\]

\[
x < 0 \quad \text{or} \quad x > 4
\]

\hspace{1cm} \text{Ans:} (-\infty, 0) \cup (4, \infty)
31. Think of the shape as 4 right triangles, a half circle, and a rectangle. The triangle has hypotenuse 4 and side 2. We use Pythagoras to find the other side:
\[ c^2 = a^2 + b^2 \]
\[ 4^2 = 2^2 + b^2 \]
\[ b \approx \sqrt{12} \approx 3.46 \]

The rectangle as width 4 and length \( 2 \times 3.46 = 6.92 \).

\[ A_{shape} = \frac{1}{2} A_{circle} + A_{rectangle} + 4A_{triangle} \]

\[ A_{circle} = \pi r^2 = 3.14 \times (2)^2 = 12.56 \]

\[ A_{rectangle} = length \times width = 6.92 \times 4 = 27.68 \]

\[ A_{triangle} = \frac{1}{2} base \times height = \frac{1}{2} \times 3.46 \times 2 = 3.46 \]

\[ A_{shape} = \frac{1}{2} \times 12.56 + 27.68 + 4 \times 3.46 = 47.80 \]

**Ans:** 47.80