Section A.7 and A.10

nth Roots, Rational Exponents, Radical Equations, & Complex Numbers

Math 1051 - Precalculus I
Solve: $3 - 5|2x - 4| < -7$
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Ans: $(-\infty, 1) \cup (3, \infty)$

$3 - 5|2x - 4| < -7$
$-5|2x - 4| < -10$
$|2x - 4| > 2$

So $2x - 4 > 2$ or $2x - 4 < -2$. 
Our first exam will be Friday during class time in this room. It will cover the Appendix.

I will do a review on Wednesday.

Be sure to check out the review sheet and exam cover sheet I will post on my web site later today.
Definitions

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Result of multiplying a number by itself
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$$n$$ is index, $$a$$ is radicand, $$\sqrt{\text{ }}$$ is radical symbol
If \( n \geq 2, m \geq 2 \) and the radicals are defined, then

\[
\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}
\]

\[
\sqrt[n]{ab} = \sqrt[n]{a^n} \cdot \sqrt[n]{b}
\]

\[
\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m
\]

\[
\text{if } n \geq 3 \text{ and } n \text{ is odd, }
\]
\[
\text{if } n \geq 2 \text{ and } n \text{ is even, }
\]

In Section A.7 and A.10, you will find detailed discussions on Roots, Exponents, and Complex Numbers.
Properties

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How to simplify radicals:

- Remove any perfect roots from the radicand
- No fractions under the radicand
- No radicals in the denominator

Examples

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Examples
Radical Equations

Put the radical on one side and then undo by exponentiating.

Solve \(3 + 2 \cdot \sqrt{x} + 4 = 5\)

Watch out! Incorrect thinking sometimes can lead to a correct solution.
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Radical Equations

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Solve $3 + 2 \cdot \sqrt{x + 4} = 5$

Watch out! Incorrect thinking sometimes can lead to a correct solution.
Solve $\sqrt{12} - x = x$
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Be sure the solution is in the domain of the original equation.
Use the rules of exponents as well as

\[ \left( \frac{a}{b} \right)^{m/n} = \frac{n \sqrt[b]{a^m}}{\sqrt[n]{b^m}} \]

Examples: Simplify

\[ 3\sqrt{x^2} \cdot \sqrt[4]{x^3} \]

\[ (4x^{1/3} - y^{1/3})^{2/3} (x^{1/3} - y^{1/3})^{3/2} \]
Rational Exponents

Use the rules of exponents as well as

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\]

\[
= \frac{\sqrt[n]{x^2} \cdot x^{1/n}}{x^{3/n}}
\]

\[
= x^{2/n} \cdot x^{1/n} \cdot x^{-3/n}
\]

\[
= x^{2/n + 1/n - 3/n}
\]

\[
= x^{-1/n}
\]

\[
= \frac{1}{\sqrt[n]{x}}
\]
Use the rules of exponents as well as

\[ a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m \]

Examples: Simplify

\[
\frac{\sqrt[3]{x^2} \cdot \sqrt{x}}{\sqrt[4]{x^3}}
\]

\[ (4x^{-1}y^{1/3})^{2/3} \]

\[ (x^{-1}y)^{3/2} \]
Imagine the solution to the equation $x^2 + 1 = 0$. 

In 1572 Italian Rafael Bombelli defined a new number: $i = \sqrt{-1}$. At the time, such numbers were regarded by some as useless. For example, René Descartes called them “imaginary” in 1637 because they appeared to be fictitious.
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Imaginary numbers are used in

- signal processing
Uses of Complex Numbers

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- cartography
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Here is a model for the propagation of a plane wave along the $x$-axis as a function of time:
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Here is a model for the propagation of a plane wave along the $x$-axis as a function of time:

$$
\psi(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \theta \left( \frac{2\pi}{\lambda} \right) e^{i \left( \frac{2\pi}{\lambda} x - \omega t \right)} d\frac{2\pi}{\lambda}
$$

where

- $\psi$ is the wave function
- $\lambda$ is the wavelength
- $\theta$ is a characteristic of the particular wave
- $\omega$ is frequency
- $t$ is time
Powers of $i$

\[ i = \sqrt{-1} \]

and so on...
Powers of $i$

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Powers of $i$


department

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Powers of \( i \)

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\[ i^5 = i^4 \cdot i = 1 \cdot i = i \]
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\[ i = \sqrt{-1} \]

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\[ i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1 \]
Powers of $i$

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\[ i^5 = i^4 \cdot i = 1 \cdot i = i \]

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and so on...
Complex numbers have the form $a + bi$ where:

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$$z = a + bi$$

$\bar{z} = a - bi$ is the complex conjugate
Operations with Complex Numbers

Tip: Add like normal and think of the $i$ as a variable like $x$.

Examples:
Operations with Complex Numbers

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Examples:

$$(2 - 3i)(4 + i)$$
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Examples:

$$(2 - 3i)(4 + i)$$

$$
\begin{array}{c}
4 \\
3i
\end{array}
$$
Operations with Complex Numbers

Tip: Add like normal and think of the $i$ as a variable like $x$.

Examples:

$$(2 - 3i)(4 + i)$$

$$\frac{4}{3i}$$

$$\frac{5}{4 + 7i}$$
Define the principal square root of a negative number as follows:
Square roots of negative numbers

Define the principal square root of a negative number as follows:

$$\sqrt{-N} = \sqrt{N}i$$
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Examples:

$$\sqrt{-4}$$

$$\sqrt{-2\sqrt{-18}}$$
Remember, at the first lecture, I proved that $1 = 2$? Let’s prove that false by proving that $1 = -1$ instead...
Start working on the review problems