Section 2.1 and 2.2

Functions, Graphs of Functions

Math 1051 - Precalculus I
Find the center and radius of the circle:

$$x^2 + y^2 + 4x + 6y + 9 = 0$$
A relation is a correspondence between two sets.

It can usually be defined by a rule, like given a number multiply it by 3 and then subtract 1.

\[ y = 3x - 1 \]
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We can also define a relation by listing our sets and using arrows to show which inputs correspond to which outputs.
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Each input has exactly one output.
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Each input has **exactly one** output.

This is a function
Functions are a special type of relation.

Each input has exactly one output.

This is a relation, but NOT a function.
Straight lines

\[ f(x) = y = 2x + 3 \]
$f(x) = y = x^2 - 1$
Sideways Parabola

\[ x = y^2 - 3 \]

This is NOT a function since one \( x \) value (\( x = 1 \)) corresponds to two different \( y \) values (\( +2 \) and \( -2 \)).

Functions, Graphs of Functions

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Sideways Parabola

\[ x = y^2 - 3 \]

This is NOT a function since one \( x \) value \((x = 1)\) corresponds to two different \( y \) values \((+2 \text{ and } -2)\).
**Vertical Line Test:** A set of points in the \( xy \)-plane is the graph of a function if and only if *every* vertical line intersects the graph in at most one point.
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Function Notation

Think of a function like a machine:
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INPUT x

FUNCTION f:

OUTPUT f(x)
\[ V = \pi r^2 h \]
If the radius is fixed, we can write this as

\[ V(h) = \pi r^2 h \]
V = \pi r^2 h

If the *height is fixed*, we can write this as

\[ V(r) = \pi r^2 h \]
If neither or fixed, we can think of this as a function of 2 variables

\[ V(r, h) = \pi r^2 h \]
\[ f(x) = -3x^2 + 2x \]
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- \( f(3) \)
- \( f(0) \)
\begin{align*}
  f(x) &= -3x^2 + 2x \\
  f(3) &\\
  f(0) &\\
  f(\text{Ralph}) &
\end{align*}
Example

If \( f(x) = 3x^2 - bx + 4 \) and \( f(-1) = 12 \), what is \( b \)?
Express the area $A$ of an isosceles right triangle as a function of the length $x$ of one of the two equal sides.
Difference Quotient: \[ \frac{f(x+h)-f(x)}{h} \]
Difference Quotient: \[ \frac{f(x+h) - f(x)}{h} \]

Find the difference quotient of \( f(x) = x^2 - 2x + 1 \).
Implicit and Explicit functions

Explicit Forms

\[ y = f(x) = -3x + 5 \]

\[ y = f(x) = x^2 - 6 \]

Implicit Forms

\[ 3x + y = 5 \]

\[ x^2 - y = 6 \]

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Implicit and Explicit functions

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Implicit Forms

- \( 3x + y = 5 \)
- \( x^2 - y = 6 \)
- \( xy = 4 \)
Domain of Functions

\[ f(x) = \frac{x + 4}{x^2 - 2x - 3} \]
Domain of Functions

\[ f(x) = \frac{x + 4}{x^2 - 2x - 3} \]
$g(x) = x^2 - 9$
Domain of Functions

\[ g(x) = x^2 - 9 \]
Domain of Functions

\[ h(x) = \sqrt{3 - 2x} \]
Domain of Functions

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$j(x) = \sqrt{2x - 1} + \frac{1}{x^2 + 5x - 6}$
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Add: \((f + g)(x) = f(x) + g(x)\)
Operations on Functions

- Add: \((f + g)(x) = f(x) + g(x)\)
- Subtract: \((f - g)(x) = f(x) - g(x)\)
Add: \((f + g)(x) = f(x) + g(x)\)

Subtract: \((f - g)(x) = f(x) - g(x)\)

Multiply: \((f \cdot g)(x) = f(x) \cdot g(x)\)
Add: \((f + g)(x) = f(x) + g(x)\)

Subtract: \((f - g)(x) = f(x) - g(x)\)

Multiply: \((f \cdot g)(x) = f(x) \cdot g(x)\)

Divide: \(\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}\)
Given: \( f(x) = \frac{2x}{x-2} \)

a. Is this point on the graph of \( f \): \((\frac{1}{2}, -\frac{2}{3})\)

b. If \( x = 4 \), what is \( f(x) \)? I.e. what is \( f(4) \)?

c. If \( f(x) = 1 \), what is \( x \)? (There may be more than one answer)

d. What is the domain of \( f \)?

e. What are the \( x \)-intercepts of \( f \)?

f. What is the \( y \)-intercept of \( f \)? (Important, for functions there can only be one. Vertical Line Test)
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Read section 2.3 for Wednesday