Sec 3.4

Quadratic Models

Math 1051 - Precalculus I
Find the vertex and axis of symmetry of

\[ f(x) = -3x^2 + 6x + 9 \]
Find the vertex and axis of symmetry of

\[ f(x) = -3x^2 + 6x + 9 \]

Ans: Vertex \((1, 12)\), Axis of Symmetry \(x = 1\)
Today...
Today...

More word problems!
The price $p$ (dollars) and the quantity $x$ sold of a certain product obey the demand equation

$$p(x) = -\frac{1}{3}x + 100$$

for $0 \leq x \leq 300$.

Express the revenue $R$ as a function of $x$
\[ R(x) = -\frac{1}{3} x^2 + 100x \]
You own a club which has 400 members who pay $500 per month in membership dues. You would like to make more money by lowering the price of membership. You make the following observation:
You own a club which has 400 members who pay $500 per month in membership dues. You would like to make more money by lowering the price of membership. You make the following observation:

For every $1 you lower the membership 2 extra people will join. What is the “optimal” price?
\[ R(p) = -2p^2 + 1400p \]
A farmer has 2000 meters of fencing. He wants to enclose a rectangular plot of land to grow corn in.
A farmer has 2000 meters of fencing. He wants to enclose a rectangular plot of land to grow corn in.

Inside he also wants to enclose a circular pond with the fence, with the diameter of the pond half the length of the shorter side of the field.
A farmer has 2000 meters of fencing. He wants to enclose a rectangular plot of land to grow corn in.

Inside he also wants to enclose a circular pond with the fence, with the diameter of the pond half the length of the shorter side of the field.

What should the length and width of the rectangle be to maximize the area for corn?
$A(x)_{\text{corn}} \approx -2x^2 + 1000x$
A window has the shape of a rectangle with an equilateral triangle on top. If the perimeter of the window is 16 feet, what dimensions will admit the most light?
A(x) \approx -x^2 + 8x
Video!
Video!

How long do you have until the cow hits you?
Here are the speeds of a Ford Taurus and its average miles per gallon. Find the speed that maximizes the miles per gallon.

<table>
<thead>
<tr>
<th>Speed</th>
<th>MPG</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>18</td>
</tr>
<tr>
<td>35</td>
<td>20</td>
</tr>
<tr>
<td>40</td>
<td>23</td>
</tr>
<tr>
<td>40</td>
<td>25</td>
</tr>
<tr>
<td>45</td>
<td>25</td>
</tr>
<tr>
<td>50</td>
<td>28</td>
</tr>
<tr>
<td>55</td>
<td>30</td>
</tr>
<tr>
<td>60</td>
<td>29</td>
</tr>
<tr>
<td>65</td>
<td>26</td>
</tr>
<tr>
<td>65</td>
<td>25</td>
</tr>
<tr>
<td>70</td>
<td>25</td>
</tr>
</tbody>
</table>
**Quadratic model:**
Ford Taurus Mileage Measurements

\[ y = -0.02x^2 + 1.9x - 25 \]
\[ R^2 = 0.89 \]

**Speed for max gas mileage:**
\[ x = \frac{-b}{2a} \approx \frac{-(1.9346)}{2(-0.0175)} \approx 55.3 \]
Linear model:

Ford Taurus Mileage Measurements

\[ y = 0.17x + 16 \quad R^2 = 0.42 \]

\[ y = -0.02x^2 + 1.9x - 25 \quad R^2 = 0.89 \]
Degree 6 polynomial model:

Ford Taurus Mileage Measurements

\[ y = 4E-07x^6 - 0.0001x^5 + 0.013x^4 - 0.84x^3 + 29x^2 - 534x + 4000 \]

\[ R^2 = 0.97 \]

\[ y = 0.17x + 16 \]
\[ R^2 = 0.42 \]

\[ y = -0.02x^2 + 1.9x - 25 \]
\[ R^2 = 0.89 \]
Flight of a Baseball
Flight of a Baseball - Effect of the density of the air
Flight of a Baseball - Effect of the density of the air

On its web page the Colorado Rockies provide the chart shown below which shows the effect of altitude on the distance a ball travels.

Distance a batted ball travels (feet) versus altitude above sea level (feet).
This table summarizes the data given in the graph.

<table>
<thead>
<tr>
<th>Stadium</th>
<th>Altitude (feet)</th>
<th>Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yankee Stadium</td>
<td>0</td>
<td>400</td>
</tr>
<tr>
<td>Turner Field</td>
<td>1050</td>
<td>408</td>
</tr>
<tr>
<td>Coors Field</td>
<td>5280</td>
<td>440</td>
</tr>
</tbody>
</table>

Linear model:
\[ D(a) = 0.00757267a + 400.02165815 \]

Quadratic model:
\[ D(a) = -0.00000001a^2 + 0.00762979a + 400 \]
This table summarizes the data given in the graph.

<table>
<thead>
<tr>
<th>Stadium</th>
<th>Altitude (feet)</th>
<th>Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yankee Stadium</td>
<td>0</td>
<td>400</td>
</tr>
<tr>
<td>Turner Field</td>
<td>1050</td>
<td>408</td>
</tr>
<tr>
<td>Coors Field</td>
<td>5280</td>
<td>440</td>
</tr>
</tbody>
</table>

Linear model: \( D(a) = 0.00757267a + 400.02165815 \)

Quadratic model: \( D(a) = -0.00000001a^2 + 0.00762979a + 400 \)
Here is the graph with linear and quadratic models:

Distance vs. Altitude Joe College HWID 101

\[ y = 0.00757267x + 400.02165815 \]
\[ R^2 = 0.99999863 \]

Distance vs. Altitude Joe College HWID 101

\[ y = -0.00000001x^2 + 0.00762979x + 400,000,000,000 \]
\[ R^2 = 1.0000000 \]
Read section 4.1 for Monday.