Sec 5.3

Exponential Functions

Math 1051 - Precalculus I
Find the inverse of

\[ f(x) = \frac{x}{x - 2} \]
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Ans:

\[ f^{-1}(x) = \frac{2x}{x - 1} \]
So far all the functions we have graphed are algebraic functions.
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All of these involve taking an unknown base $x$ to various powers/exponents like $x^{1/2} = \sqrt{x}$.

We can instead fix the base and let the exponent change. For example, $f(x) = 2^x$
What does the function $f(x) = 2^x$ look like? We can get an idea by plotting points.
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Let’s find some of the properties of this function.
A general exponential function has the form $f(x) = c \cdot a^x$
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- where \( a \) is some fixed number
- \( a > 0 \)
- \( a \neq 1 \)
- \( c \) is some constant that stretches the output by a fixed amount
Exponential functions follow the laws of exponents:
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- \( a^s \cdot a^t = a^{s+t} \)
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- \( (ab)^s = a^s b^s \)
- \( a^0 = 1 \)
Once we know the graph of $f(x) = 2^x$ we can use it to graph other exponential functions using transformations.
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Graph $f(x) = -2^{-(x-4)} + 3$
Once we know the graph of \( f(x) = 2^x \) we can use it to graph other exponential functions using transformations.

Graph \( f(x) = -2^{-(x-4)} + 3 \)
Graphs of different exponential functions

\[ f(x) = 2^x \]
Graphs of different exponential functions

\[ f(x) = 4^x \]
Graphs of different exponential functions

$f(x) = 10^x$
How about $f(x) = \left(\frac{1}{2}\right)^x$?
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We can start with $2^x$ and use transformations.
Exponential Functions

Sec 5.3

\[ f(x) = 2^x \]
Exponential Functions

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\[ f(x) = \left( \frac{1}{2} \right)^x \]
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\[ f(x) = \left( \frac{1}{4} \right)^x \]
Inverse of an exponential function

Exponential Functions
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\[ f(x) = 2^x \]
Inverse of an exponential function

\[ f^{-1}(x) \]
These involve terms of the form $a^x$
Exponential Equations

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For example,

$$3^{2x-6} = 81^{x-2}$$
Exponential Equations

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For example,

$$3^{2x-6} = 81^{x-2}$$
The equation $3^{2x-6} = 81^{x-2}$ is shown graphically, with a table comparing $3^{2x-6}$ to $81^{x-2}$ for various values of $x$. The values for $x = 2.0$ to $x = 1.0$ are compared, showing the relationship between the two expressions over the interval.
Solve $3^{2x-6} = 81^{x-2}$

The important rule is:

If $a^x = a^u$, then $x = u$
Leonhard Euler (1707-1783)

\[ e = \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^n \]
Leonhard Euler (1707-1783)

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<th>( \left(1 + \frac{1}{n}\right)^n )</th>
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</table>
Leonhard Euler (1707-1783)

\[ e = 2.7182818284590 \ldots \]

Euler's Identity:

\[ e^{i\pi} = -1 \]
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e = 2.7182818284590...
Leonhard Euler (1707-1783)

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Euler’s Identity:

\[ e^{i\pi} = -1 \]
Solve: \((e^4)^x e^{x^2} = e^{12}\)
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If \(e^{-2x} = 2\), what does \(e^{x-1}\) equal?
Read section 5.4 for Friday.