Exam 2 Review

Covers chapters 1 and 2

Math 1051 - Precalculus I
A rectangle is inscribed in an isosceles right triangle whose hypotenuse lies along the $x$-axis and is 8 units long. Express the area $A$ of the rectangle in terms of $x$. 

Ans: $A(x) = -2x^2 + 8x$
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Ans: $A(x) = -2x^2 + 8x$
Sec 1.1 Distance and Midpoint Formulas

Distance Formula
\[ d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Midpoint Formula
\[ x_{\text{midpoint}} = \frac{x_1 + x_2}{2} \]
\[ y_{\text{midpoint}} = \frac{y_1 + y_2}{2} \]

These are the average of the coordinates.

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These are the average of the coordinates.
Sec 1.2 Graphs in Two Variables

Graph equations by plotting points.

- **x-intercept** is the value of $x$ when $y = 0$.
- **y-intercept** is the value of $y$ when $x = 0$.

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Sec 1.3 Linear Relations

Slope-intercept form:
\[ y = mx + b \]

Point-slope form:
\[ y - y_1 = m(x - x_1) \]

General form:
\[ Ax + By = C \]

Average rate of change:
\[ m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}} \]

Horizontal lines:
\[ y = k \]

Vertical lines:
\[ x = k \]

Parallel lines: Same slope

Perpendicular lines: Slopes are negative reciprocals
\[ m_1 = -\frac{1}{m_2} \]

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- Perpendicular lines: Slopes are negative reciprocals

\[ m_1 = -\frac{1}{m_2} \]
Find the equation of a line perpendicular to the line 
$3x - y = -4$ and which passes through $(-2, 4)$. Write the 
answer in slope-intercept form and general form.
Sec 1.4 Circles

Standard form of a circle equation:

\[(x - h)^2 + (y - k)^2 = r^2\]

General form of a circle equation:

\[x^2 + y^2 + ax + by + c = 0\]

Find the center and radius of the circle described by

\[2x^2 + 2y^2 - 4x + 8y = 0\]
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Sec 2.1 Functions

A function can be thought of as a machine...

If \( f(x) = -3x^2 + 1 \), what is \( f(x+h) \)?
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If \( f(x) = -3x^2 + 1 \), what is \( f(x + h) \)?
Domain: Allowable values of $x$ in a function
Sec 2.1 Functions

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Find the domain of

$$f(x) = \frac{\sqrt{x + 6}}{|x + 6| - 2}$$
Sec 2.3 Properties of Functions

Even function:
$f(-x) = f(x)$, symmetric about y-axis

Odd function:
$f(-x) = -f(x)$, symmetric about origin

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Sec 2.3 Properties of Functions

Even function: \( f(-x) = f(x) \),

Odd function: \( f(-x) = -f(x) \), symmetric about the origin.
Even function: $f(-x) = f(x)$, symmetric about $y$-axis
Sec 2.3 Properties of Functions

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Sec 2.3 Properties of Functions

Even function: \( f(-x) = f(x) \), symmetric about \( y-axis \)

Odd function: \( f(-x) = -f(x) \), symmetric about \( origin \)
Is \( f(x) = \frac{x^2 - 2}{x^3 - 3x} \) even, odd, or neither?
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Average Rate of change from $c$ to $x$ is

$$ARC = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}$$
Average Rate of change from \( c \) to \( x \) is

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Find the average rate of change of \( f(x) = 3x^2 - 1 \) from 1 to 2 and the secant line that connect \((1, f(1))\) and \((2, f(2))\).
Basic functions

- Square Root: \( f(x) = \sqrt{x} \)
- Cube Root: \( f(x) = \sqrt[3]{x} \)
- Identity: \( f(x) = x \)
- Square: \( f(x) = x^2 \)
- Cube: \( f(x) = x^3 \)
- Constant: \( f(x) = b \)
- Absolute value: \( f(x) = |x| \)
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- **Identity**: \( f(x) = x \)
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- **Cube**: \( f(x) = x^3 \)
- **Constant**: \( f(x) = b \)
- **Absolute value**: \( f(x) = |x| \)
Reciprocal:

\[ f(x) = \frac{1}{x} \]
- Reciprocal:
  \[ f(x) = \frac{1}{x} \]

- Reciprocal Square:
  \[ f(x) = \frac{1}{x^2} \]
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• Reciprocal Square:
  \[ f(x) = \frac{1}{x^2} \]

• Greatest Integer:
  \[ f(x) = \text{int}(x) = \lfloor x \rfloor \]
Graph

\[ f(x) = \begin{cases} 
  x^2 + 1 & \text{if } -3 < x < 2 \\
  x + 3 & \text{if } 3 \leq x \leq 5 \\
  -4 & \text{if } x > 5 
\end{cases} \]
Piecewise Functions

Graph

\[
f(x) = \begin{cases} 
  x^2 + 1 & \text{if } -3 < x < 2 \\
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Tip: Plot some points within each domain
Piecewise Functions

Graph

\[ f(x) = \begin{cases} 
  x^2 + 1 & \text{if } -3 < x < 2 \\
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\end{cases} \]

Tip: Plot some points within each domain

Things you may need to know: domain, range, x-intercepts, y-intercept
Graph $f(x) = -3(x - 1)^3 + 2$ using transformations.
Page 97 in the textbook has a nice summary if this is still confusing.
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RCS
A rectangle is inscribed in a circle of radius 8 centered at the origin. Let $P = (x, y)$ be the point in quadrant 1 that is a vertex of the rectangle and is on the circle. Express the area $A$ of the rectangle as a function of $x$. 
Have a nice weekend!