The diagnostic test is designed to give us an idea of your level of skill in doing high school algebra as you begin Math 1051. You should be able to do these problems correctly and without too much difficulty. Even though Math 1051 begins with a brief review of high school algebra, if you are really lost doing these problems you should talk to Professor Robertson (612-625-1075, droberts@umn.edu) immediately.

Go through the solutions and be sure you understand them. Then, do the homework problems given in the boxes and hand them in at the second lecture session.

On the papers you turn in for grading, be sure to print your name, your 3-digit homework ID number, and HW3 at the top of the first page.

1. Simplify: \[ 8 - \frac{2(3) - 4(-3)}{6(-2) - (-3)} \]

Follow the order of operations for simplifying arithmetic expressions:

Step 1 Simplify expressions inside grouping symbols, which include parentheses ( ), brackets [ ], the fraction bar \( \frac{a}{b} \), absolute value bars \( | \) , and radicals \( \sqrt[n]{a} \).

Step 2 Simplify exponents, square roots, and absolute values.

Step 3 Simplify multiplication and division, working left to right.

Step 4 Simplify addition and subtraction, working left to right.

\[ 8 - \frac{2(3) - 4(-3)}{6(-2) - (-3)} = 8 - \frac{6 + 12}{-12 + 3} = 8 - \frac{18}{-9} = 8 - (-2) = 8 + 2 = 10 \]

Homework: Simplify each of the following. Check the answers at the end of this document.

1a. \[ 1 + 4^2 \div 2 - (7 - 2) \cdot 3 \]
1b. \[ \sqrt{20 - 4} + 6^2 \div (5 - 1) - 2 \cdot (11 - 3) \]
1c. \[ 6 - \frac{7 - 2^3 \cdot 9}{18 \div 2 \cdot 3 - 7 \cdot 2} \]

2. Simplify: \[ \frac{1}{10} + \frac{1}{12} - \frac{5}{18} \] (For more information see Sullivan pages A37 - A39.)

Step 1 Find the Least Common Denominator (LCD) by prime factoring each denominator and then multiplying the largest group of each prime factor.

\[ 10 = 2 \cdot 5 \]
\[ 12 = 2 \cdot 2 \cdot 3 \]
\[ 18 = 2 \cdot 3 \cdot 3 \]

Largest group of 2's is 2 \cdot 2 (in 12)
Largest group of 3's is 3 \cdot 3 (in 18)
Largest group of 5's is 5 (in 10)

So, the LCD = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180
**Step 2** Convert each given fraction into an equivalent fraction with the LCD as the denominator.

\[
\begin{align*}
\frac{1}{10} + \frac{1}{12} &- \frac{5}{18} \\
\frac{1}{10} \cdot \frac{18}{18} + \frac{1}{12} \cdot \frac{15}{15} &- \frac{5}{10} \cdot \frac{18}{18} \\
\frac{18}{180} + \frac{15}{180} &- \frac{50}{180}
\end{align*}
\]

**Step 3** Now that the fractions have the same denominator, add or subtract the numerators.

\[
\frac{18 + 15 - 50}{180} = \frac{-17}{180}
\]

**Step 4** Reduce by canceling factors common to the numerator and denominator.

\[
\frac{-17}{180} = \frac{-17}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}
\]

Since there are no factors common to the numerator and denominator the fraction cannot be reduced. The final answer is \(-\frac{17}{180}\).

**Homework:** Simplify each of the following. Check the answers at the end of this document.

<table>
<thead>
<tr>
<th></th>
<th>2a. (\frac{9}{10} - \frac{1}{9} + \frac{14}{15})</th>
<th>2b. (-\frac{7}{15} + \frac{5}{12} + \frac{7}{10})</th>
<th>2c. (\frac{3}{4} + \frac{1}{9} - \frac{13}{36})</th>
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3. Simplify: \(\sqrt{50x^{16}y^9}\) where \(x \geq 0, y \geq 0\) (For more information see Sullivan, pages A82 - A83.)

Factor the radicand. Each pair of identical factors is a perfect square and so that factor will come out from under the radical. Any left-over single factors will remain under the radical.

\[
\sqrt{50x^{16}y^9} = \sqrt{2 \cdot 5 \cdot 5 \cdot x^8 \cdot x^8 \cdot x \cdot x \cdot x \cdot y^3 \cdot y^3 \cdot y^3 \cdot y^3 \cdot y^3}
\]

\[
= 5 \cdot x^8 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \sqrt{2y}
\]

\[
= 5x^8y^4\sqrt{2y}
\]

Rather than writing out all the factors of \(x\) and \(y\), we could simply divide their exponents by the index of the radical, which is 2 since we have a square root. The resulting quotient is the exponent of the variable outside the radical and the resulting remainder is the exponent of the variable under the radical.

\[
\begin{align*}
x^{16} &\rightarrow 2^{16} & 8 \leftarrow \text{Exponent of } x \text{ outside radical} \\
x^{16} &\rightarrow 2^{16} & 16 \leftarrow \text{Exponent of } y \text{ outside radical} \\
0 &\leftarrow \text{No } x \text{'s remain under radical} \\
0 &\leftarrow \text{No } y \text{'s remain under radical}
\end{align*}
\]

**Homework:** Simplify each of the following. Check the answers at the end of this document.

<table>
<thead>
<tr>
<th></th>
<th>3a. (\sqrt{72x^{25}y^7}) where (x \geq 0, y \geq 0)</th>
<th>3b. (\frac{3}{54}x^{8}y^{27})</th>
<th>3c. (\frac{5}{128}x^9y^{34})</th>
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4. Factor: $2x^2 + 5x - 12$ (For more information see Sullivan pages A28 - A29.)

We are going to do a LOT of factoring in this course. Here is a general procedure to follow:

**Step 1** Factor out the Greatest Common Factor (GCF).

For example, factor: $6x^3y^5 + 8xy^4 + 2xy^3$

$$6x^3y^5 + 8xy^4 + 2xy^3 = 2 \cdot 3 \cdot xxx \cdot yyyyy + 2 \cdot 2 \cdot 2 \cdot x \cdot yyy + 2 \cdot x \cdot yyy$$

$$= 2 \cdot 3 \cdot xxx \cdot yyyyy + 2 \cdot 2 \cdot 2 \cdot x \cdot yyy + 2 \cdot x \cdot yyy$$

$$= 2 \cdot x \cdot yyy (3xxxyy + 2 \cdot 2y + 1)$$

$$= 2xy^3 (3x^2y^2 + 4y + 1)$$

The GCF is $2xy^3$. Notice the third term, 1, must be there for the factorization to be correct.

**Step 2** Count the number of terms and look for factoring patterns.

**Two terms:** Try factoring using one of these patterns (*memorize these!*)

- Difference of perfect squares: $x^2 - a^2 = (x - a)(x + a)$
- Difference or sum of perfect cubes: $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

Notice the only difference between the above two formulas are the signs.

**Three terms:**

- Try factoring using the patterns for perfect square trinomials: $x^2 + 2ax + a^2 = (x + a)^2$

Notice the only difference between the above two formulas are the signs.

- If the trinomial has the form $x^2 + bx + c$, find two integers whose product is $c$ and whose sum is $b$. Let’s say the integers are $m$ and $n$. Then $x^2 + bx + c = (x + m)(x + n)$.

For example, factor: $x^2 + x - 12$

We need two integers whose product is $-12$ and whose sum is $1$. Integer factors of $12$ are $1 \cdot 12, 2 \cdot 6, \text{ and } 3 \cdot 4$. Since the product is negative, the two factors must have different signs. Since the sum must be $1$, we choose $-3 \cdot 4$.

Thus, $x^2 + x - 12 = (x - 3)(x + 4)$.

- If the trinomial has the form $ax^2 + bx + c$, find two integers whose product is $a \cdot c$ and whose sum is $b$, replace the $bx$ term using these integers, and factor by grouping. For an example, see below.
Four terms: Try factoring by grouping.

For example, factor: $5x^2 + 15x - 2xy - 6y$

Group the first pair of terms and factor out the GCF; do the same for the second pair of terms.

$$5x^2 + 15x - 2xy - 6y = (5x^2 + 15x) + (-2xy - 6y)$$
$$= 5x(x + 3) + (-2y)(x + 3)$$
$$= (x + 3)(5x - 2y)$$

Step 3 Factor completely. In some cases, it may be necessary to factor more than once. Remember that multiplication can be used to check the factorization.

Now, let's factor $2x^2 + 5x - 12$:

Step 1 GCF: There is no factor common to all the terms so there is no GCF other than 1 or $-1$.

Step 2 Number of terms: There are three terms and they match the pattern $ax^2 + bx + c$, where $a = 2$, $b = 5$, and $c = -12$. So, we find two integers whose product is $a \cdot c = 2 \cdot (-12) = -24$ and whose sum is $b = 5$. Here are the possibilities for 24 (ignore the sign for the moment):

$$24 = 1 \cdot 24, 2 \cdot 12, 3 \cdot 8, 4 \cdot 6$$

The pair that can have a sum of 5 is $3 \cdot 8$ if we attach a negative sign to the 3. The integers we seek are $-3$ and $8$ since $(-3)(8) = -24$ and $-3 + 8 = 5$.

Replace the middle term, $5x$, with its equivalent $-3x + 8x$ and factor by grouping:

$$2x^2 + 5x - 12 = 2x^2 - 3x + 8x - 12$$
$$= (2x^2 - 3x) + (8x - 12)$$
$$= x(2x - 3) + 4(2x - 3)$$
$$= (2x - 3)(x + 4)$$

Step 3 Factor completely: Since the expression cannot be further factored the answer is $(2x - 3)(x + 4)$.

Homework: Factor each of the following. Check the answers at the end of this document.

| 4a. $4x^2 - 64x + 256$ | 4b. $6x^2 - x - 12$ | 4c. $4x^2 - 20x + 25$ |
5. Simplify: $\frac{(3x^3)^2 y^5}{9(xy^3)^{-2}}$ where $x \neq 0, y \neq 0$ (For more information see Sullivan, pages A7 - A9)

Be sure you have memorized the laws of exponents:

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n} \text{ where } a \neq 0$$

$$(a^m)^n = a^{mn}$$

$$(ab)^m = a^m b^m$$

$$a^{-m} = \frac{1}{a^m} \text{ where } a \neq 0$$

$$a^0 = 1 \text{ where } a \neq 0$$

It usually is easiest to simplify the numerator and denominator individually and then simplify the quotient.

$$\frac{(3x^3)^2 y^5}{9(xy^3)^{-2}} = \frac{3^{2}x^{6}y^{5}}{9x^{1(-2)}y^{3(-2)}} = \frac{3^{2}x^{6}y^{5}}{9x^{-2}y^{-6}} = \frac{9x^{6}y^{5}}{9x^{-2}y^{-6}} = x^{6-(-2)}y^{5-(-6)} = x^{8}y^{11}$$

**Homework:** Simplify each of the following. Assume neither $x$ nor $y$ are 0. Check the answers at the end of this document.

5a. $\left(2x^2\right)^3 \left(3x^4\right)^2$  
5b. $\frac{36x^3y^{12}}{10x^5y^{-3}}$ where $x \neq 0, y \neq 0$  
5c. $\frac{2x^{-2}\left(3y^4\right)^2}{6x^{-3}y^5}$ where $x \neq 0, y \neq 0$
6. Solve: \( \frac{x}{3} - \frac{3x - 1}{6} = \frac{5x + 16}{12} \) (For more information see Sullivan pages A43 - A45.)

When solving an equation that contains fractions, it usually is easiest if you clear the fractions first by multiplying each term by the LCD of all the terms. The LCD of 3, 6, and 12 is 12, so multiply each term by 12 and then simplify.

\[
\begin{align*}
12 \cdot \left[ \frac{x}{3} \right] - 12 \cdot \left[ \frac{3x - 1}{6} \right] &= 12 \cdot \left[ \frac{5x + 16}{12} \right] \\
4 \cdot \frac{x}{12} - 2 \cdot \frac{3x - 1}{6} &= \frac{1}{12} \cdot \frac{5x + 16}{12} \\
4x - 2(3x - 1) &= 5x + 16 \\
4x - 6x + 2 &= 5x + 16 \\
-2x + 2 &= 5x + 16 \\
-7x &= 14 \\
x &= -2
\end{align*}
\]

**Homework:** Solve each of the following. Check the answers at the end of this document.

6a. \( 2(x + 1) = 6(x - 3) - 7x + 11 \)  
6b. \( \frac{1}{3}x + \frac{1}{2}(1 - x) = 4 \)  
6c. \( \frac{x - 2}{6} - \frac{1}{8}(x + 2) = \frac{-7 + x}{12} \)

**Answers to homework problems:**

1a. –6  
1b. –3  
1c. 11  
2a. \( \frac{31}{18} \)  
2b. \( \frac{13}{20} \)  
2c. \( \frac{1}{2} \)  
3a. \( 6x^{12}y^3\sqrt{2xy} \)  
3b. \( 3x^2y^9 \cdot 3\sqrt{2x^2} \)  
3c. \( 2xy^6 \cdot 5\sqrt{4x^4y^4} \)  
4a. \( 4(x - 8)(x - 8) \)  
4b. \( (3x + 4)(2x - 3) \)  
4c. \( (2x - 5)(2x - 5) \)  
5a. \( 72x^{14} \)  
5b. \( \frac{18y^{15}}{5x^2} \)  
5c. \( 3xy^3 \)  
6a. \( x = -3 \)  
6b. \( x = -21 \)  
6c. \( x = 0 \)