Midterm 1 Review

Note: This is not intended to be your only study guide. Please refer to class notes, homework problems, and previous quizzes as well.

1. Find the equation of the plane that contains the points $(1, 2, 1)$, $(2, 0, 2)$ and $(-1, 0, 1)$.

2. Define a vector $\mathbf{u} = (-1, 2, -1)$.

   (a) Find a unit vector that is parallel to $\mathbf{u}$.

   (b) Find a unit vector that is perpendicular to $\mathbf{u}$.

3. Identify and sketch the graph of the surface $x^2 = y^2 + 4z^2$.

4. Find and sketch the domain of the function $f(x, y) = \sqrt{4 - x^2 - y^2}$.

5. Evaluate $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2 + 2y^2}$ or show it does not exist.

6. Find all second partial derivatives of $v = x\cos(y + 2z)$

7. Find the linear approximation of the function $f(x, y, z) = x^3\sqrt{y^2 + z^2}$ at the point (2,3,4).

8. Consider the surface $x^2 + 2y^2 - 3z^2 = 3$.

   (a) Find the equation of the tangent plane to the surface.

   (b) Find the equation of the normal line to the surface.

9. If $u = x^2y^3 + z^4$, where $x = p + 3p^2, y = pe^p$, and $z = p\sin(p)$, use the Chain Rule to find $\frac{du}{dp}$.

10. Let $f(x, y) = x^2y + \sqrt{y}$.

    (a) Find the directional derivative of f at the point $(2, 1)$ in the direction $(1, 1)$.

    (b) Find the maximum rate of change of f at the point $(2, 1)$. In which direction does it occur?

11. Find the local maximum and minimum values and saddle points of the function $f(x, y) = 3xy - x^2y - xy^2$.

12. Find the absolute maximum and minimum values of $f(x, y) = 4xy^2 - x^2y^2 - xy^3$ on the closed triangular region with vertices $(0, 0), (0, 6)$, and $(6, 0)$.

13. Find the maximum and minimum values of $f(x, y, z) = x^2y$ subject to the constraint $x^2 + y^2 = 1$.

14. Calculate the iterated integral $\int_1^2 \int_0^2 (y + 2xe^y)dxdy$